W pair production at the LHC
I. Virtual $\mathcal{O}(\alpha_s^2)$ corrections in the high energy limit

G. Chachamis,[a] **M. Czakon**[ab] **and D. Eiras**[a]

[a]*Institut für Theoretische Physik und Astrophysik, Universität Würzburg,*
Am Hubland, D-97074 Würzburg, Germany
[b]*Department of Field Theory and Particle Physics,*
Institute of Physics, University of Silesia,
Uniwersytecka 4, PL-40007 Katowice, Poland
E-mail: chachamis@physik.uni-wuerzburg.de, mczakon@physik.uni-wuerzburg.de,
Dolores.Eiras@physik.uni-wuerzburg.de

ABSTRACT: We present the result for the two-loop and the one-loop squared virtual QCD corrections to the W boson pair production in the quark-anti-quark-annihilation channel in the limit where all kinematical invariants are large compared to the mass of the W boson. The infrared pole structure is in agreement with the prediction of Catani's general formalism for the singularities of two loop amplitudes.

JHEP12(2008)003

Contents

1. Introduction

The Large Hadron Collider (LHC) will be the centre of interest for particle physics phenomenology in the next years. Open issues that require definite answers are the verification of the consistency and validity of the Standard Model (SM) in the energy range of the LHC as well as insights into New Physics. Several proposed models and concepts that have the SM as their low energy limit theory are either to pass the LHC test or to be proven wrong. Supersymmetry and Extra-dimensions are two of the most illustrious examples.

Probably, the most important goal for the LHC is the discovery of the elusive Higgs boson. The latter is part of the mechanism of dynamical breaking of the Electroweak (EW) symmetry and is responsible for the fermions and gauge bosons mass. Discovering the only constituent of the Standard Model (SM) which has not been experimentally observed yet, along with a systematic measurement of its properties, will be essential for our understanding of mass and the precise gauge structure of the SM. Another important endeavour at the LHC, in connection to the investigation of the non-Abelian gauge structure of the SM, is the precise measurement of the hadronic production of gauge boson pairs, $WW, WZ, ZZ, W\gamma$, $Z\gamma$. Deviations from the SM predictions would indicate the presence of either anomalous couplings or new heavy particles which would decay into vector boson pairs [1, 2].

Seen under the prism of the previous argumentation, W pair production via quark-anti-quark-annihilation,

$$q\bar{q} \to W^+ W^- , \tag{1.1}$$

is a very important process at the LHC. Firstly, it can serve as a signal process in the search for New Physics since it can be used to measure the vector boson trilinear couplings as

predicted by the Standard Model (SM) (actually, this is the favored channel as it involves both trilinear vertices, WWZ and $WW\gamma$). Secondly, $q\bar{q} \to W^+W^-$ is the dominant irreducible background to the promising Higgs discovery channel

$$pp \to H \to W^*W^* \to l\bar{\nu}l'\nu' \,, \tag{1.2}$$

in the mass range M_{Higgs} between 140 and 180 GeV [3].

Due to its importance, the study of W pair production in hadronic collisions has attracted a lot of attention in the literature. The Born cross section was calculated almost 30 years ago [4], whereas the next-to-leading order (NLO) QCD corrections to the tree-level were computed in refs. [5 – 9] and were proven to be large. They enhance the tree-level by almost 70% which falls to a (still) large 30% after imposing a jet veto. Therefore, if a theoretical estimate for the W pair production is to be compared against experimental measurements at the LHC, one is bound to go one order higher in the perturbative expansion, namely to the next-to-next-to-leading order (NNLO). This would allow, in principle, an accuracy of better than 10%. Notice that first steps in this direction have been done by considering soft-gluon resummation effects in W pair production [10].

High accuracy for the W pair production is also needed when the process is studied as background to Higgs production. The NLO QCD corrections to the signal process for the Higgs discovery via gluon fusion, $gg \to H$, contribute a 70% [11, 12], whereas the NNLO contributions suggest an additional 20% for the LHC [13 – 15]. With a jet veto, at NNLO the total corrections are of the order of 85% [16 – 18]. Lastly, the QCD corrections to the cross section for the process $H \to WW \to l\bar{\nu}l'\nu'$ are known at NNLO [19, 20] whereas the EW ones are known beyond NLO [21]. The ratio of the Higgs signal over background is expected between 1:1 and 2:1 once certain cuts are applied that reject events with high p_T jets. For a consistent QCD analysis, therefore, we need to compare both signal and background cross sections calculated at the same order, that is, at NNLO. Another process that needs to be included in the background is the W pair production in the loop induced gluon fusion channel,

$$gg \to W^+W^- \,. \tag{1.3}$$

This contributes at $\mathcal{O}(\alpha_s^2)$ relative to the quark-anti-quark-annihilation channel but is nevertheless enhanced due to the large gluon flux at the LHC. The corrections from gluon fusion increase the W pair background estimate by almost 30% after certain experimental Higgs search cuts are imposed [22, 23].

In this paper, we address the task of computing the NNLO virtual part, more precisely the interference of the two-loop with the Born amplitude, as well as the the one-loop squared contribution. We work in the limit of fixed scattering angle and high energy, where all kinematical invariants are large compared to the mass m of the W. Our result contains all logarithms $\log m$ as well as all constant contributions while we neglect power corrections in m. These will be presented in a following publication.

Our methodology for obtaining the massive amplitude (massless fermion-boson scattering was studied in ref. [24]) is very similar to the one followed in refs. [25 – 27] which is, at its turn, an evolution of the methods employed in refs. [28, 29]. The amplitude is

reduced to an expression that only contains a small number of integrals (master integrals) with the help of the Laporta algorithm [30]. In the calculation for the two-loop amplitude there are 71 master integrals. For the one-loop squared case, we use the helicity matrix formalism to reduce the problem to a small set of integrals. Next comes the construction, in a fully automatised way, of the Mellin-Barnes (MB) representations [31, 32] of all the master integrals by using the `MBrepresentation` package [33]. The representations are then analytically continued in the number of space-time dimensions by means of the `MB` package [34], thus revealing the full singularity structure. An asymptotic expansion in the mass parameter is performed by closing contours and the integrals are finally resummed, either with the help of `XSummer` [35] or the `PSLQ` algorithm [36].

Our paper is organised as follows. In section 2 we introduce our notation, present briefly our methods and define the perturbative expansion of the matrix elements summed over colours and spins. In section 3 we study the singular behavior of the NNLO contributions, and verify that it agrees with the general formalism developed by Catani [37] for the infrared structure of QCD amplitudes. In section 4 we present the finite remainders for the interference of the tree and the two-loop amplitude and the one-loop squared after subtraction of the singular poles of section 3 from the explicit result. We organise the finite part according to the colour content of the two-loop amplitude for the two-loop case. The finite remainders are expressed in terms of logarithms and polylogarithms which are real in the physical domain. We conclude in section 5. Finally, for completeness, the one-loop result up to order ϵ^2 is included in the appendix.

2. Notation

The charged vector-boson production in the leading partonic scattering process corresponds to

$$q_j(p_1) + \overline{q}_j(p_2) \;\; \rightarrow \;\; W^-(p_3, m) + W^+(p_4, m) \,, \tag{2.1}$$

where p_i denote the quark and W momenta, m is the mass of the W boson and j is a flavour index. We are considering down type quark scattering in our paper. Obtaining the corresponding result for up-type quark scattering is actually trivial as we will show in the following. Energy-momentum conservation implies

$$p_1^\mu + p_2^\mu = p_3^\mu + p_4^\mu \,. \tag{2.2}$$

We consider the scattering amplitude $\mathcal{M}$ for the process (2.1) at fixed values of the external parton momenta p_i, thus $p_1^2 = p_2^2 = 0$ and $p_3^2 = p_4^2 = m^2$. The amplitude $\mathcal{M}$ may be written as a series expansion in the strong coupling $\alpha_{\rm s}$,

$$|\mathcal{M}\rangle = \left[|\mathcal{M}^{(0)}\rangle + \left(\frac{\alpha_{\rm s}}{2\pi} \right) |\mathcal{M}^{(1)}\rangle + \left(\frac{\alpha_{\rm s}}{2\pi} \right)^2 |\mathcal{M}^{(2)}\rangle + \mathcal{O}(\alpha_{\rm s}^3) \right] \,, \tag{2.3}$$

and we define the expansion parameter in powers of $\alpha_{\rm s}(\mu^2)/(2\pi)$ with μ being the renormalisation scale. We work in conventional dimensional regularisation, $d = 4 - 2\epsilon$, in the $\overline{\rm MS}$-scheme for the coupling constant renormalisation.

We explicitly relate the bare (unrenormalised) coupling $\alpha_{\rm s}^{\rm b}$ to the renormalised coupling $\alpha_{\rm s}$ by

$$\alpha_{\rm s}^{\rm b} S_\epsilon = \alpha_{\rm s}\left[1 - \frac{\beta_0}{\epsilon}\left(\frac{\alpha_{\rm s}}{2\pi}\right) + \mathcal{O}(\alpha_{\rm s}^2)\right], \qquad (2.4)$$

where we set the factor $S_\epsilon = (4\pi)^\epsilon \exp(-\epsilon\,\gamma_{\rm E}) = 1$ for simplicity and β is the QCD β-function known at present up to the four-loop level [38, 39]

$$\beta_0 = \frac{11}{6}C_A - \frac{2}{3}T_F n_f\,. \qquad (2.5)$$

The color factors in a non-Abelian SU(N)-gauge theory are $C_A = N$, $C_F = (N^2 - 1)/2N$ and $T_F = 1/2$. Throughout this paper, N denotes the number of colors and n_f the total number of flavors of massless quarks. Remark, however, that the latter must come in pairs, because of the flavor changing coupling to the charged gauge boson. This is only problematic in the case of top quarks running in a closed loop.

In the following, our discussion will be restricted to the two-loop amplitude summed over spins and colours and contracted with the Born one. Nevertheless, it should be stressed that our methods and the results of the present work can be easily extended to the partial amplitudes for the individual helicity combinations of the massive two-loop amplitude $|\mathcal{M}^{(2)}\rangle$ itself.

For convenience, we define the function $\mathcal{A}(\epsilon, m, s, t, \mu)$ for the squared amplitudes summed over spins and colors as

$$\overline{\sum |\mathcal{M}(q_j + \overline{q}_j \to W^+ + W^-)|^2} = \mathcal{A}(\epsilon, m, s, t, \mu)\,. \qquad (2.6)$$

$\mathcal{A}$ is a function of the Mandelstam variables s, t and u given by

$$s = (p_1 + p_2)^2\,, \qquad t = (p_1 - p_3)^2 - m^2\,, \qquad u = (p_1 - p_4)^2 - m^2\,, \qquad (2.7)$$

and has a perturbative expansion similar to eq. (2.3),

$$\mathcal{A}(\epsilon, m, s, t, \mu) = \left[\mathcal{A}^{(0)} + \left(\frac{\alpha_{\rm s}}{2\pi}\right)\mathcal{A}^{(1)} + \left(\frac{\alpha_{\rm s}}{2\pi}\right)^2\mathcal{A}^{(2)} + \mathcal{O}(\alpha_{\rm s}^3)\right]\,. \qquad (2.8)$$

In terms of the amplitudes the expansion coefficients in eq. (2.8) may be expressed as

$$\mathcal{A}^{(0)} = \langle\mathcal{M}^{(0)}|\mathcal{M}^{(0)}\rangle\,, \qquad (2.9)$$

$$\mathcal{A}^{(1)} = \left(\langle\mathcal{M}^{(0)}|\mathcal{M}^{(1)}\rangle + \langle\mathcal{M}^{(1)}|\mathcal{M}^{(0)}\rangle\right)\,, \qquad (2.10)$$

$$\mathcal{A}^{(2)} = \left(\langle\mathcal{M}^{(1)}|\mathcal{M}^{(1)}\rangle + \langle\mathcal{M}^{(0)}|\mathcal{M}^{(2)}\rangle + \langle\mathcal{M}^{(2)}|\mathcal{M}^{(0)}\rangle\right)\,, \qquad (2.11)$$

where $\mathcal{M}^{(0)}$ and $\mathcal{M}^{(1)}$ are the massive tree level and one loop amplitudes correspondingly. $\mathcal{A}^{(0)}$ is given by

$$\mathcal{A}^{(0)} = N\left\{c_1\left[16(1-\epsilon)^2\frac{x}{(1-x)} + 4(3-4\epsilon)\frac{1}{m_s} + \frac{4x(1-x)}{m_s^2}\right]\right.$$

$$+ c_2 \left[-24 + 16x + 16\epsilon\,(2-x) + 4\frac{(3-4\epsilon) - 2x(1-x)}{m_s} + \frac{4x\,(1-x)}{m_s^2} \right]$$
$$+ c_3 \left[-24\,(1-x(1-x)) + 16\epsilon(2-x(1-x)) + \frac{6-8\epsilon-8x(1-x)}{m_s} + \frac{2x(1-x)}{m_s^2} \right] \right\}, \quad (2.12)$$

where we have defined $x = -\frac{t}{s}$, $m_s = \frac{m^2}{s}$ and only the leading physical powers (i.e. down to the constant) in the m_s-expansion are retained. Notice that, once the actual values of the c_i are substituted, the terms singular in m_s cancel as required by unitarity. This will be the case for the final two-loop and one-loop squared expressions as well. The coefficients c_1, c_2 and c_3 are in their essence combinations of EW coupling constants defined as

$$c_1 = \frac{g_{WL}^4}{4},$$
$$c_2 = \frac{1}{4\,s_w^2} \left(Q_q + 2g_{ZL}^q \frac{c_w}{s_w \left(1 - \frac{M_Z^2}{s}\right)} \right),$$
$$c_3 = \frac{c_w^2}{s_w^2 (1 - \frac{M_Z^2}{s})^2} \left((g_{ZA}^q)^2 + \left(g_{ZV}^q + Q_q \frac{s_w \left(1 - \frac{M_Z^2}{s}\right)}{c_w} \right)^2 \right). \quad (2.13)$$

In order to compute the $\langle \mathcal{M}^{(1)} | \mathcal{M}^{(1)} \rangle$ it proves convenient to express the amplitude $|\mathcal{M}^{(1)}\rangle$ in terms of helicity amplitudes, $\mathcal{M}^g(\lambda_1, \lambda_2, s, t)$, where λ_1 and λ_2 stand for the helicities of the W^+ and W^- respectively. In other words, it is convinient to decompose the amplitude into a sum of products consisting generally of three parts, a Feynman integral, a rational function of the kinematical variables and a standard matrix element, $\mathcal{M}_j^g$, a complete list of which is listed below in eq. (2.15). For that, Dirac algebra is used, as well as the equations of motion and an anticommuting γ_5. The quark and anti-quark have opposite helicities in the centre-of-mass system so one helicity label above, $g = \pm 1$, suffices.

Therefore, the one-loop amplitude is formally rearranged as

$$|\mathcal{M}^{(1)}\rangle = \sum_{i,j,g} C_i(s,t,u)\mathcal{I}_i^j(s,t,u;\mu^2)\mathcal{M}_j(\{p_k\},g), \quad (2.14)$$

where the C_i are coefficients, the $\mathcal{I}_i^j$ are one-loop dimensionally regularized scalar integrals, $\mathcal{M}_j$ are helicity matrix elements, $\mathrm{g} = \pm$ and $k = 1, \ldots, 4$. The ten helicity matrix elements $\mathcal{M}_j(p_k, g) = \mathcal{M}_j^g$ have been taken as defined in ref. [40] (see also [41]):

$$\mathcal{M}_0^g = \overline{v}(p_2)\,\not{\varepsilon}_1(\not{p}_3 - \not{p}_2)\not{\varepsilon}_2 \mathcal{P}_g\,u(p_1),$$
$$\mathcal{M}_1^g = \overline{v}(p_2)\,\not{p}_3 \mathcal{P}_g\,u(p_1)\,\varepsilon_1 \cdot \varepsilon_2,$$
$$\mathcal{M}_2^g = \overline{v}(p_2)\,\not{\varepsilon}_1 \mathcal{P}_g\,u(p_1)\,\varepsilon_2 \cdot p_3,$$
$$\mathcal{M}_3^g = -\overline{v}(p_2)\,\not{\varepsilon}_2 \mathcal{P}_g\,u(p_1)\,\varepsilon_1 \cdot p_4,$$
$$\mathcal{M}_4^g = \overline{v}(p_2)\,\not{\varepsilon}_1 \mathcal{P}_g\,u(p_1)\,\varepsilon_2 \cdot p_1,$$
$$\mathcal{M}_5^g = -\overline{v}(p_2)\,\not{\varepsilon}_2 \mathcal{P}_g\,u(p_1)\,\varepsilon_1 \cdot p_2, \quad (2.15)$$
$$\mathcal{M}_6^g = \overline{v}(p_2)\,\not{p}_3 \mathcal{P}_g\,u(p_1)\,\varepsilon_1 \cdot p_2\,\varepsilon_2 \cdot p_1,$$

$$\mathcal{M}_7^g = \overline{v}(p_2)\,\not{p}_3\mathcal{P}_g\,u(p_1)\,\varepsilon_1\cdot p_2\,\varepsilon_2\cdot p_3\,,$$

$$\mathcal{M}_8^g = \overline{v}(p_2)\,\not{p}_3\mathcal{P}_g\,u(p_1)\,\varepsilon_1\cdot p_4\,\varepsilon_2\cdot p_1\,,$$

$$\mathcal{M}_9^g = \overline{v}(p_2)\,\not{p}_3\mathcal{P}_g\,u(p_1)\,\varepsilon_1\cdot p_4\,\varepsilon_2\cdot p_3\,,$$

where $\mathcal{P}_g = \mathcal{P}_\pm = \frac{1\pm\gamma_5}{2}$. All colour indices as well as the arguments of the polarization vectors, $\varepsilon_1(p_3,\lambda_1)$ and $\varepsilon_2(p_4,\lambda_2)$, have been suppressed.

After expressing the one-loop amplitude as in eq. (2.14) and calculating the Feynman integrals, it is trivial to obtain $\langle\mathcal{M}^{(1)}|\mathcal{M}^{(1)}\rangle$, one needs only to compute the traces (one fermionic chain in all cases) coming from multipling a matrix element with the complex conjugate of another one. We have decomposed the tree level amplitude as well in terms of helicity amplitudes and computed $\langle\mathcal{M}^{(0)}|\mathcal{M}^{(1)}\rangle$ as a trivial cross check. Even though the representations in eq. (2.15) have been used internally, here we present our result only for the amplitude squared and summed over helicities. Notice that this is done in conventional dimensional regularization, which implies $2-2\epsilon$ polarizations of the vector bosons.

The expressions for $\mathcal{A}^{(1)}$ have been presented e.g. in refs. [5, 6] whereas the leading color coefficient of $\langle\mathcal{M}^{(0)}|\mathcal{M}^{(2)}\rangle$ was discussed in ref. [42]. Here we provide for the first time the result for the real part of $\mathcal{A}^{(2)}$.

3. Infrared pole structure

In the simpler case of one-loop amplitudes, their poles in ϵ can be expressed as a universal combination of the tree amplitude and a colour-charge operator $\mathrm{I}^{(1)}(\epsilon)$. The generic form of the $\mathrm{I}^{(1)}(\epsilon)$ operator was found by Catani and Seymour [45] (see also [43, 44]) and it was derived for the general one-loop QCD amplitude by integrating the real radiation graphs of the same order in perturbation series in the one-particle unresolved limit.

The pole structure of our one-loop expression is given, according to the prediction by Catani, by acting with the operator $\mathrm{I}^{(1)}(\epsilon)$ onto the tree-level result:

$$|\mathcal{M}^{(1)}\rangle = \mathrm{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle + |\mathcal{M}_{\text{finite}}^{(1)}\rangle\,, \tag{3.1}$$

where $\mathrm{I}^{(1)}(\epsilon)$ is defined as

$$\mathrm{I}^{(1)}(\epsilon) = -C_F\frac{e^{\epsilon\gamma}}{\Gamma(1-\epsilon)}\left(\frac{1}{\epsilon^2}+\frac{3}{2\epsilon}\right)\left(-\frac{\mu^2}{s}\right)^\epsilon\,. \tag{3.2}$$

In a similar way, the divergences of the two-loop amplitude can be written as a sum of two terms: the action of the $\mathrm{I}^{(1)}(\epsilon)$ operator on the one-loop amplitude and the action of a new operator $\mathrm{I}^{(2)}(\epsilon)$ on the tree amplitude. The $\mathrm{I}^{(2)}(\epsilon)$ operator includes a renormalisation scheme dependent term $H^{(2)}$ multiplied by a $1/\epsilon$ pole. In the following, we give explicit expressions for $\mathrm{I}^{(1)}(\epsilon)$ and $\mathrm{I}^{(2)}(\epsilon)$ which are valid in the $\overline{\text{MS}}$ scheme.

At next-to-next-to-leading-order (NNLO), contributions from the self-interference of the one-loop amplitude and the interference of the tree and the two-loop amplitude must be taken into account, so that

$$\mathcal{A}^{\text{NNLO}}(s,t,u,m,\mu) = \mathcal{A}^{\text{NNLO}(1\times1)}(s,t,u,m,\mu) + \mathcal{A}^{\text{NNLO}(0\times2)}(s,t,u,m,\mu), \tag{3.3}$$

with

$$\mathcal{A}^{\text{NNLO}(1\times1)}(s,t,u,m,\mu) = \langle\mathcal{M}^{(1)}|\mathcal{M}^{(1)}\rangle, \tag{3.4}$$

and

$$\mathcal{A}^{\text{NNLO}(0\times2)}(s,t,u,m,\mu) = \langle\mathcal{M}^{(0)}|\mathcal{M}^{(2)}\rangle + \langle\mathcal{M}^{(2)}|\mathcal{M}^{(0)}\rangle. \tag{3.5}$$

We further decompose the one-loop self-interference and the two-loop contributions as a sum of singular and finite terms,

$$\mathcal{A}^{\text{NNLO}(1\times1)}(s,t,u,m,\mu) = \mathcal{C}^{(1\times1)}_{\text{atani}}(s,t,u,m,\mu) + \mathcal{F}^{(1\times1)}_{inite}(s,t,u,m,\mu) \tag{3.6}$$

and

$$\mathcal{A}^{\text{NNLO}(0\times2)}(s,t,u,m,\mu) = \mathcal{C}^{(0\times2)}_{\text{atani}}(s,t,u,m,\mu) + \mathcal{F}^{(0\times2)}_{inite}(s,t,u,m,\mu), \tag{3.7}$$

$\mathcal{C}^{(1\times1)}_{\text{atani}}$ and $\mathcal{C}^{(0\times2)}_{\text{atani}}$ contain infrared singularities that will be analytically canceled by the infrared singularities occurring in radiative processes of the same order (ultraviolet divergences having already been removed by renormalisation). $\mathcal{F}^{(1\times1)}_{inite}$ and $\mathcal{F}^{(0\times2)}_{inite}$ are the remainders which are finite as $\epsilon \to 0$.

The infrared poles of the interference of the tree and the two-loop amplitudes follow a generic formula developed by Catani in ref. [37]. Due to the simple colour structure of the process (2.1) the action of $\text{I}^{(1)}(\epsilon)$ and $\text{I}^{(2)}(\epsilon)$ is factorised such that we formally have

$$\mathcal{C}^{(1\times1)}_{\text{atani}}(s,t,u,m,\mu) = |\text{I}^{(1)}(\epsilon)|^2\langle\mathcal{M}^{(0)}|\mathcal{M}^{(0)}\rangle + 2\text{Re}\left\{\text{I}^{(1)}(\epsilon)^*\langle\mathcal{M}^{(0)}|\mathcal{M}^{(1)}_{\text{finite}}\rangle\right\}. \tag{3.8}$$

and

$$\mathcal{C}^{(0\times2)}_{\text{atani}}(s,t,u,m,\mu) = 2\text{Re}\left\{\text{I}^{(1)}(\epsilon)\langle M^{(0)}|M^{(1)}\rangle + \text{I}^{(2)}(\epsilon)\langle M^{(0)}|M^{(0)}\rangle\right\} \tag{3.9}$$

with

$$\text{I}^{(2)}(\epsilon) = -\frac{1}{2}\text{I}^{(1)}(\epsilon)\left(\text{I}^{(1)}(\epsilon) + \frac{2\beta_0}{\epsilon}\right) + \frac{e^{-\epsilon\gamma}\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{\beta_0}{\epsilon} + K\right)\text{I}^{(1)}(2\epsilon)$$
$$+ H^{(2)}(\epsilon), \tag{3.10}$$

where

$$K = \left(\frac{67}{18} - \frac{\pi^2}{6}\right)C_A - \frac{10}{9}T_F n_f. \tag{3.11}$$

The renormalisation scheme dependent $H^{(2)}$ constant for a QCD amplitude with a $q\bar{q}$ pair is given by

$$H^{(2)}(\epsilon) = 2\frac{e^{\epsilon\gamma}}{4\epsilon\Gamma(1-\epsilon)}\left(-\frac{\mu^2}{s}\right)^{2\epsilon}\left\{\left(\frac{\pi^2}{2} - 6\,\zeta_3 - \frac{3}{8}\right)C_F^2\right.$$
$$\left. + \left(\frac{13}{2}\zeta_3 + \frac{245}{216} - \frac{23}{48}\pi^2\right)C_A C_F + \left(-\frac{25}{54} + \frac{\pi^2}{12}\right)C_F T_F n_f\right\}. \tag{3.12}$$

We were able to verify that our results have the same infrared structure as the one predicted by Catani's formalism.

4. Results

4.1 Two-loop contribution

In this section, we give explicit expressions for the finite remainder of the two-loop contribution $\mathcal{F}_{inite}^{(0\times2)}$ defined as

$$\mathcal{F}_{inite}^{(0\times2)}(s,t,u,m,\mu) = \mathcal{A}^{\text{NNLO}\,(0\times2)}(s,t,u,m,\mu) - \mathcal{C}_{\text{atani}}^{(0\times2)}(s,t,u,m,\mu)\,, \qquad (4.1)$$

or in the rescaled form

$$\mathcal{F}_{inite}^{(0\times2)}\left(m_s,x,\frac{s}{\mu^2}\right) = \mathcal{A}^{\text{NNLO}\,(0\times2)}\left(m_s,x,\frac{s}{\mu^2}\right) - \mathcal{C}_{\text{atani}}^{(0\times2)}\left(m_s,x,\frac{s}{\mu^2}\right)\,. \qquad (4.2)$$

The EW structure of the finite remainder for a down-type quark can be factorised as

$$\mathcal{F}_{inite,\,down}^{(0\times2)} = 2\,N\sum_{i=1,4} c_i\mathcal{J}_{i,\,down}^{(1\times1)}\left(m_s,x,\frac{s}{\mu^2}\right)\,. \qquad (4.3)$$

This decomposition allows one to easily obtain the result for the up-type quark scattering. The latter is then given by

$$\mathcal{F}_{inite,\,up}^{(0\times2)} = 2\,N\sum_{i=1,4} c_i\mathcal{J}_{i,\,up}^{(0\times2)}\left(m_s,x,\frac{s}{\mu^2}\right)\,, \qquad (4.4)$$

where one needs to use the following formulae

$$\mathcal{J}_{1,\,up}^{(0\times2)}\left(m_s,x,\frac{s}{\mu^2}\right) = \mathcal{J}_{1,\,down}^{(0\times2)}\left(m_s,y,\frac{s}{\mu^2}\right)\,, \qquad (4.5)$$

$$\mathcal{J}_{2,\,up}^{(0\times2)}\left(m_s,x,\frac{s}{\mu^2}\right) = -\mathcal{J}_{2,\,down}^{(0\times2)}\left(m_s,y,\frac{s}{\mu^2}\right)\,, \qquad (4.6)$$

$$\mathcal{J}_{3,\,up}^{(0\times2)}\left(m_s,x,\frac{s}{\mu^2}\right) = \mathcal{J}_{3,\,down}^{(0\times2)}\left(m_s,y,\frac{s}{\mu^2}\right)\,, \qquad (4.7)$$

$$\mathcal{J}_{4,\,up}^{(0\times2)}\left(m_s,x,\frac{s}{\mu^2}\right) = \mathcal{J}_{4,\,down}^{(0\times2)}\left(m_s,y,\frac{s}{\mu^2}\right) \qquad (4.8)$$

and naturally to make the corresponding changes in the definitions of the couplings c_1, c_2, c_3 and c_4, namely to use the up-type quark charge and isospin. Here $y = -\frac{u}{s}$. In the following and with no loss of clarity, since our result assumes down-type quark scattering, we will suppress all indices that indicate the type of scattered quark. The functions $\mathcal{J}_i(m_s,x,\frac{s}{\mu^2})$ in eq. (4.3) will be presented decomposed according to the colour structure, namely in the form

$$\mathcal{J}_i^{(0\times2)}\left(m_s,x,\frac{s}{\mu^2}\right) = \left(j_i^{(1)}C_FC_A + j_i^{(2)}C_F^2 + j_i^{(3)}C_FT_Fn_f\right)\,. \qquad (4.9)$$

c_4, in addition to c_1, c_2 and c_3, is a new coupling that appears at the two-loop level and is defined as:

$$c_4 = -\frac{c_w\,g_{ZA}^q}{2\,s_w^3(1-\frac{M_Z^2}{s})}\,. \qquad (4.10)$$
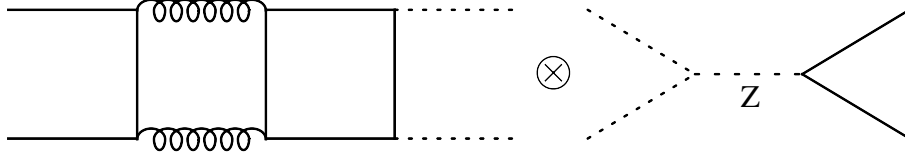
Figure 1: Born diagram with a Z exchanged in the s-channel contracted with a fermionic two-loop box.

The appearance of c_4 is an effect that comes from a specific part of $\langle \mathcal{M}^{(0)} | \mathcal{M}^{(2)} \rangle$. This part consists of two-loop fermionic boxes contracted with the Born diagram that involves an s-channel Z exchange. A typical example can be seen in figure (1). The main feature of these diagrams is that their EW couplings fall into two disjoint fermionic chains and once the traces are computed the axial part drops out. By adding and subtracting to the surviving vector part the corresponding axial part, one can combine vector and axial contributions into a piece proportional to c_2. The remaining piece is proportional to what we have defined as c_4.

We have verified that applying the naive recipe of sending all traces that contain a single γ_5 independently to zero is a valid approach for this class of diagrams. We did this by calculating explicitly the output after substituting γ_5 by its alternative form $\gamma_5 = \frac{i}{4!}\epsilon_{\mu\nu\alpha\beta}\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\beta$ and confirming that no additional terms survive. This was in fact a non-trivial cancellation as it occurs only for the sum of all the diagrams of this particular class. Note, however, that there were finite contributions from traces containing γ_5 in the case of pure W boson pair exchange (no photons or Z's involved).

We are finally ready to present our result. The functions $\mathcal{J}_i$, are given by

$$\mathcal{J}_1^{(0\times2)}(m_s, x) =$$
$$C_A C_F \left\{ \frac{1}{m_s{}^2} \left[\frac{31}{120}(1-x)x\pi^4 - \frac{107}{36}(1-x)x\pi^2 - \frac{51157}{648}(1-x)x + \frac{659}{18}(1-x)x\zeta_3 + \frac{88}{3}(1-x)x\mathrm{L}_s \right] \right.$$
$$+ \frac{1}{m_s} \left[\frac{31\pi^4}{40} - \frac{107\pi^2}{12} + \frac{659\zeta_3}{6} + 88\mathrm{L}_s - \frac{51157}{216} \right] + \left[\frac{1}{30}\left(-684x^3 + 684x^2 - 114x + \frac{31}{1-x} - 31\right)\pi^4 \right.$$
$$+ \frac{1}{9}\left(1404x^3 - 1404x^2 + 188x - \frac{303}{1-x} + 359 - \frac{108}{x}\right)\pi^2 - \frac{8}{3}\left(2 - \frac{1}{1-x}\right)\mathrm{Li}_2(x)\pi^2 + \frac{1}{3}\left(6x^3 - 6x^2 + x\right)\mathrm{L}_x^4$$
$$+ \frac{1}{3}\left(6x^3 - 6x^2 + x - 2\right)\mathrm{L}_y^4 - \frac{4}{3}\left(36x^3 - 30x^2 + 11x - \frac{3}{1-x} + 3\right)\mathrm{L}_x^3 - \frac{8}{3}\left(6x^3 - 6x^2 + x\right)\mathrm{L}_m\mathrm{L}_x^3$$
$$+ \frac{4}{9}\left(-108x^3 + 126x^2 - 33x - \frac{22}{1-x} + 43 - \frac{15}{x} + \frac{9}{x^2}\right)\mathrm{L}_y^3 - \frac{8}{3}\left(6x^3 - 6x^2 + x\right)\mathrm{L}_m\mathrm{L}_y^3$$
$$+ \frac{4}{3}\left(6x^3 - 6x^2 + x + 2\right)\mathrm{L}_x\mathrm{L}_y^3 + \left(4\left(6x^3 - 6x^2 + x\right)\pi^2 + 24x\right)\mathrm{L}_m^2 + 4\left(6x^3 - 6x^2 + x\right)\mathrm{L}_m^2\mathrm{L}_x^2$$
$$+ \left(4\left(39x^3 - 15x^2 + x - \frac{4}{1-x} + 7\right) - \frac{14}{3}\left(6x^3 - 6x^2 + x\right)\pi^2\right)\mathrm{L}_x^2 - 8\left(6x^3 - 6x^2 + x\right)\mathrm{Li}_2(x)\mathrm{L}_x^2$$
$$+ 4\left(36x^3 - 24x^2 + 7x - \frac{3}{1-x} + 3\right)\mathrm{L}_m\mathrm{L}_x^2 + 4\left(6x^3 - 6x^2 + x\right)\mathrm{L}_m^2\mathrm{L}_y^2 - 2\left(6x^3 - 6x^2 + x + \frac{2}{1-x}\right)\mathrm{L}_x^2\mathrm{L}_y^2$$
$$+ \left(-\frac{2}{3}\left(42x^3 - 42x^2 + 7x + \frac{2}{1-x} + 2\right)\pi^2 - \frac{2}{9}\left(-702x^3 + 1134x^2 - 378x - \frac{167}{1-x} + 185 - \frac{144}{x} + \frac{108}{x^2}\right)\right)\mathrm{L}_y^2$$
$$+ 4\left(36x^3 - 48x^2 + 15x - 5 + \frac{5}{x} - \frac{3}{x^2}\right)\mathrm{L}_m\mathrm{L}_y^2 + \frac{44}{3}\left(1 - \frac{1}{1-x}\right)\mathrm{L}_s\mathrm{L}_y^2 + 8\left(3x^2 - x\right)\mathrm{L}_x\mathrm{L}_y^2$$

$$+\frac{1}{162}\left(-15480x+\left(-46656x^3+46656x^2-9072x+\frac{19836}{1-x}-15948\right)\zeta_3-\frac{58897}{1-x}+58897\right)$$

$$-8\left(36x^3-36x^2+15x-\frac{4}{1-x}+5\right)\mathrm{Li}_3(x)-16\left(6x^3-6x^2+x\right)\mathrm{Li}_4(x)+\left(4\left(36x^3-36x^2+11x-1\right)\pi^2\right.$$

$$\left.-\frac{164x}{9}+\left(-96x^3+96x^2-16x\right)\zeta_3+\frac{596}{9(1-x)}-\frac{1888}{9}\right)\mathrm{L}_m-16\left(6x^3-6x^2+x\right)\mathrm{Li}_3(x)\mathrm{L}_m$$

$$-\frac{44}{3}\left(-2x-\frac{9}{1-x}+9\right)\mathrm{L}_s+\frac{88}{3}\left(x-\frac{1}{1-x}+2\right)\mathrm{L}_m\mathrm{L}_s-16\left(3x^2-2x\right)\mathrm{L}_m^2\mathrm{L}_x$$

$$+\left(8\left(7x^2-2x-\frac{2}{1-x}+3\right)\pi^2-24\left(5x^2-2x-1\right)\right)\mathrm{L}_x+8\left(36x^3-36x^2+15x-\frac{4}{1-x}+5\right)\mathrm{Li}_2(x)\mathrm{L}_x$$

$$+16\left(6x^3-6x^2+x\right)\mathrm{Li}_3(x)\mathrm{L}_x-8\left(24x^2-16x+\frac{2}{1-x}+1\right)\mathrm{L}_m\mathrm{L}_x+16\left(6x^3-6x^2+x\right)\mathrm{Li}_2(x)\mathrm{L}_m\mathrm{L}_x$$

$$-\frac{4}{3}\left(6x^3-6x^2+x\right)\mathrm{L}_x^3\mathrm{L}_y+16\left(3x^2-2x\right)\mathrm{L}_m^2\mathrm{L}_y+4\left(36x^3-42x^2+17x-\frac{3}{1-x}+3\right)\mathrm{L}_x^2\mathrm{L}_y$$

$$+8\left(6x^3-6x^2+x\right)\mathrm{L}_m\mathrm{L}_x^2\mathrm{L}_y+\left(120x^2-\frac{272x}{3}-\frac{4}{9}\left(126x^2-36x+\frac{22}{1-x}+14-\frac{45}{x}+\frac{27}{x^2}\right)\pi^2\right.$$

$$\left.+\left(80-\frac{64}{1-x}\right)\zeta_3+\frac{694}{9(1-x)}-\frac{478}{9}-\frac{24}{x}\right)\mathrm{L}_y+\frac{16\mathrm{Li}_3(x)\mathrm{L}_y}{1-x}+\frac{8}{3}\left(72x^2-55x-\frac{11}{1-x}+37-\frac{9}{x}\right)\mathrm{L}_m\mathrm{L}_y$$

$$+\frac{44}{3}\left(1-\frac{1}{1-x}\right)\mathrm{L}_s\mathrm{L}_y-8\left(6x^3-6x^2+x\right)\mathrm{L}_m^2\mathrm{L}_x\mathrm{L}_y+\left(-\frac{4}{3}\left(-42x^3+42x^2-7x-\frac{2}{1-x}+4\right)\pi^2\right.$$

$$\left.-8\left(39x^3-39x^2+8x-\frac{2}{1-x}+1\right)\right)\mathrm{L}_x\mathrm{L}_y-\frac{16\mathrm{Li}_2(x)\mathrm{L}_x\mathrm{L}_y}{1-x}-8\left(36x^3-36x^2+11x-1\right)\mathrm{L}_m\mathrm{L}_x\mathrm{L}_y$$

$$-8\left(-36x^3+36x^2-7x-\frac{2}{1-x}+3+\frac{5}{x}-\frac{3}{x^2}\right)\mathrm{S}_{1,2}(x)+16\left(6x^3-6x^2+x\right)\mathrm{L}_m\mathrm{S}_{1,2}(x)-\frac{16\mathrm{L}_x\mathrm{S}_{1,2}(x)}{1-x}$$

$$\left.\left.-16\left(2-\frac{1}{1-x}\right)\mathrm{L}_y\mathrm{S}_{1,2}(x)-16\left(-6x^3+6x^2-x-\frac{3}{1-x}+5\right)\mathrm{S}_{1,3}(x)+\frac{16\mathrm{S}_{2,2}(x)}{1-x}\right]\right\}$$

$$+{C_F}^2\left\{\frac{1}{m_s^2}\left[-\frac{11}{45}(1-x)x\pi^4+\frac{29}{6}(1-x)x\pi^2+\frac{255}{8}(1-x)x-30(1-x)x\zeta_3\right]\right.$$

$$+\frac{1}{m_s}\left[-\frac{11}{15}\pi^4+\frac{29\pi^2}{2}-90\zeta_3+\frac{765}{8}\right]+\left[-\frac{2}{45}\left(-1026x^3+1026x^2-171x+\frac{44}{1-x}\right)\pi^4\right.$$

$$-\frac{2}{3}\left(468x^3-468x^2+54x-\frac{57}{1-x}+59-\frac{24}{x}\right)\pi^2+16\mathrm{Li}_2(x)\pi^2-\frac{2}{3}\left(6x^3-6x^2+x\right)\mathrm{L}_x^4$$

$$-\frac{2}{3}\left(6x^3-6x^2+x-\frac{3}{1-x}+1\right)\mathrm{L}_y^4+\frac{8}{3}\left(36x^3-30x^2+11x-\frac{3}{1-x}+3\right)\mathrm{L}_x^3+\frac{16}{3}\left(6x^3-6x^2+x\right)\mathrm{L}_m\mathrm{L}_x^3$$

$$-\frac{4}{3}\left(-72x^3+84x^2-24x-\frac{3}{1-x}+11-\frac{12}{x}+\frac{4}{x^2}\right)\mathrm{L}_y^3+\frac{16}{3}\left(6x^3-6x^2+x\right)\mathrm{L}_m\mathrm{L}_y^3$$

$$-\frac{8}{3}\left(6x^3-6x^2+x+2\right)\mathrm{L}_x\mathrm{L}_y^3+\left(-8\left(6x^3-6x^2+x\right)\pi^2-4\left(12x-\frac{1}{1-x}+1\right)\right)\mathrm{L}_m^2$$

$$-8\left(6x^3-6x^2+x\right)\mathrm{L}_m^2\mathrm{L}_x^2+\left(\frac{28}{3}\left(6x^3-6x^2+x\right)\pi^2-8\left(39x^3-15x^2+x-\frac{4}{1-x}+7\right)\right)\mathrm{L}_x^2$$

$$+16\left(6x^3-6x^2+x\right)\mathrm{Li}_2(x)\mathrm{L}_x^2-8\left(36x^3-24x^2+7x-\frac{3}{1-x}+3\right)\mathrm{L}_m\mathrm{L}_x^2-8\left(6x^3-6x^2+x\right)\mathrm{L}_m^2\mathrm{L}_y^2$$

$$+4\left(6x^3-6x^2+x+\frac{2}{1-x}\right)\mathrm{L}_x^2\mathrm{L}_y^2+\left(\frac{4}{3}\left(42x^3-42x^2+7x-\frac{6}{1-x}+10\right)\pi^2\right.$$

$$\left.+2\left(-156x^3+252x^2-88x-\frac{19}{1-x}+17-\frac{16}{x}+\frac{16}{x^2}\right)\right)\mathrm{L}_y^2$$

$$+8\left(-36x^3+48x^2-16x+\frac{1}{1-x}+3-\frac{6}{x}+\frac{2}{x^2}\right)\mathrm{L}_m\mathrm{L}_y^2-16\left(3x^2-x\right)\mathrm{L}_x\mathrm{L}_y^2$$

$$+\frac{1}{2}\left(136x+\left(1152x^3-1152x^2+224x-\frac{192}{1-x}+96\right)\zeta_3+\frac{331}{1-x}-331\right)$$

$$+16\left(36x^3-36x^2+15x-\frac{4}{1-x}+5\right)\mathrm{Li}_3(x)+32\left(6x^3-6x^2+x\right)\mathrm{Li}_4(x)$$

$$+\left(-8\left(36x^3-36x^2+11x-1\right)\pi^2-36x+\left(192x^3-192x^2+32x\right)\zeta_3-\frac{52}{1-x}+192\right)\mathrm{L}_m$$

$$+32\left(6x^3-6x^2+x\right)\mathrm{Li}_3(x)\mathrm{L}_m+32\left(3x^2-2x\right)\mathrm{L}_m^2\mathrm{L}_x$$

$$+\left(48\left(5x^2-2x-1\right)-16\left(7x^2-2x-\frac{2}{1-x}+3\right)\pi^2\right)\mathrm{L}_x-16\left(36x^3-36x^2+15x-\frac{4}{1-x}+5\right)\mathrm{Li}_2(x)\mathrm{L}_x$$

$$-32\left(6x^3-6x^2+x\right)\mathrm{Li}_3(x)\mathrm{L}_x+16\left(24x^2-16x+\frac{2}{1-x}+1\right)\mathrm{L}_m\mathrm{L}_x-32\left(6x^3-6x^2+x\right)\mathrm{Li}_2(x)\mathrm{L}_m\mathrm{L}_x$$

$$+\frac{8}{3}\left(6x^3-6x^2+x\right)\mathrm{L}_x^3\mathrm{L}_y-32\left(3x^2-2x\right)\mathrm{L}_m^2\mathrm{L}_y-8\left(36x^3-42x^2+17x-\frac{3}{1-x}+3\right)\mathrm{L}_x^2\mathrm{L}_y$$

$$-16\left(6x^3-6x^2+x\right)\mathrm{L}_m\mathrm{L}_x^2\mathrm{L}_y+\left(-240x^2+232x+\frac{8}{3}\left(42x^2-13x-\frac{6}{1-x}+13-\frac{18}{x}+\frac{6}{x^2}\right)\pi^2\right.$$

$$\left.+\left(\frac{80}{1-x}-112\right)\zeta_3-\frac{50}{1-x}+10+\frac{32}{x}\right)\mathrm{L}_y-\frac{32\mathrm{Li}_3(x)\mathrm{L}_y}{1-x}-8\left(48x^2-41x-\frac{2}{1-x}+15-\frac{4}{x}\right)\mathrm{L}_m\mathrm{L}_y$$

$$+16\left(6x^3-6x^2+x\right)\mathrm{L}_m^2\mathrm{L}_x\mathrm{L}_y+\left(\frac{8}{3}\left(-42x^3+42x^2-7x-\frac{2}{1-x}+4\right)\pi^2\right.$$

$$\left.+16\left(39x^3-39x^2+8x-\frac{2}{1-x}+1\right)\right)\mathrm{L}_x\mathrm{L}_y+\frac{32\mathrm{Li}_2(x)\mathrm{L}_x\mathrm{L}_y}{1-x}+16\left(36x^3-36x^2+11x-1\right)\mathrm{L}_m\mathrm{L}_x\mathrm{L}_y$$

$$-8\left(72x^3-72x^2+16x-\frac{1}{1-x}+1-\frac{12}{x}+\frac{4}{x^2}\right)\mathrm{S}_{1,2}(x)-32\left(6x^3-6x^2+x\right)\mathrm{L}_m\mathrm{S}_{1,2}(x)+\frac{32\mathrm{L}_x\mathrm{S}_{1,2}(x)}{1-x}$$

$$\left.\left.+16\left(3-\frac{1}{1-x}\right)\mathrm{L}_y\mathrm{S}_{1,2}(x)+16\left(-12x^3+12x^2-2x-\frac{3}{1-x}+7\right)\mathrm{S}_{1,3}(x)+32\mathrm{S}_{2,2}(x)\right]\right\}$$

$$+n_f T_F C_F\left\{\frac{1}{m_s^2}\left[\frac{7}{9}(1-x)x\pi^2+\frac{4085}{162}(1-x)x-\frac{2}{9}(1-x)x\zeta_3-\frac{32}{3}(1-x)x\mathrm{L}_s\right]\right.$$

$$+\frac{1}{m_s}\left[\frac{7\pi^2}{3}-\frac{2\zeta_3}{3}-32\mathrm{L}_s+\frac{4085}{54}\right]+\left[-\frac{44}{45}\left(1-\frac{2}{1-x}\right)\pi^4-\frac{4}{9}\left(80x+\frac{12}{1-x}+22-\frac{27}{x}\right)\pi^2\right.$$

$$+\frac{16}{3}\left(1-\frac{2}{1-x}\right)\mathrm{Li}_2(x)\pi^2+\frac{4}{3}\left(2x^2-2x-\frac{4}{1-x}+\frac{3}{(x-1)^2}+1\right)\mathrm{L}_x^3$$

$$-\frac{4}{9}\left(6x^2-6x-\frac{8}{1-x}+11-\frac{12}{x}+\frac{9}{x^2}\right)\mathrm{L}_y^3-4\left(8x-\frac{10}{1-x}+\frac{3}{(x-1)^2}+4\right)\mathrm{L}_x^2$$

$$-4\left(\frac{3}{(x-1)^2}+1-\frac{4}{1-x}\right)\mathrm{L}_m\mathrm{L}_x^2+\frac{4}{9}\left(-72x-\frac{26}{1-x}+152-\frac{117}{x}+\frac{54}{x^2}\right)\mathrm{L}_y^2+4\left(1-\frac{4}{x}+\frac{3}{x^2}\right)\mathrm{L}_m\mathrm{L}_y^2$$

$$-\frac{16}{3}\left(1-\frac{1}{1-x}\right)\mathrm{L}_s\mathrm{L}_y^2-\frac{4}{3}\left(4x^2-2x+1\right)\mathrm{L}_x\mathrm{L}_y^2-\frac{2}{81}\left(-1368x\right.$$

$$\left.+\left(1080x^2-2160x-\frac{720}{1-x}-\frac{972}{(x-1)^2}+2664\right)\zeta_3-\frac{4769}{1-x}+4769\right)$$

$$+\frac{8}{3}\left(10x^2-8x-\frac{7}{1-x}-\frac{9}{(x-1)^2}+19\right)\mathrm{Li}_3(x)+\left(\frac{16}{9}\left(13x-\frac{13}{1-x}+32\right)-4(2x-1)\pi^2\right)\mathrm{L}_m$$

$$+\frac{16}{3}\left(-2x-\frac{9}{1-x}+9\right)\mathrm{L}_s-\frac{32}{3}\left(x-\frac{1}{1-x}+2\right)\mathrm{L}_m\mathrm{L}_s-16\left(x^2-x\right)\mathrm{L}_m^2\mathrm{L}_x$$

$$+\left(-\frac{4}{9}\left(32x^2-28x-\frac{41}{1-x}+\frac{27}{(x-1)^2}+32\right)\pi^2-\frac{8}{3}\left(44x^2-18x-9\right)\right)\mathrm{L}_x$$

$$-\frac{8}{3}\left(10x^2-8x-\frac{7}{1-x}-\frac{9}{(x-1)^2}+19\right)\mathrm{Li}_2(x)\mathrm{L}_x-8\left(16x^2-14x+\frac{3}{1-x}\right)\mathrm{L}_m\mathrm{L}_x+16\left(x^2-x\right)\mathrm{L}_m^2\mathrm{L}_y$$

$$-4\left(2x^2-2x-\frac{1}{1-x}-\frac{3}{(x-1)^2}+4\right)\mathrm{L}_x^2\mathrm{L}_y+\left(\frac{4}{9}\left(32x^2-16x+\frac{8}{1-x}+12-\frac{36}{x}+\frac{27}{x^2}\right)\pi^2\right.$$

$$\left.+\frac{8}{9}\left(132x^2-198x-\frac{31}{1-x}+58+\frac{27}{x}\right)\right)\mathrm{L}_y-\frac{8}{3}\left(-48x^2+58x-\frac{4}{1-x}+2-\frac{9}{x}\right)\mathrm{L}_m\mathrm{L}_y$$

$$-\frac{16}{3}\left(1-\frac{1}{1-x}\right)L_sL_y+\frac{8}{3}(20x-3)L_xL_y+8(2x-1)L_mL_xL_y+\frac{8}{3}\left(10x^2-8x-11+\frac{12}{x}-\frac{9}{x^2}\right)S_{1,2}(x)$$
$$+32\left(1-\frac{2}{1-x}\right)S_{2,2}(x)\Big]\Big\}\,,\tag{4.11}$$

$$\mathcal{J}_2^{(0\times2)}(m_s,x)=$$
$$C_AC_F\Bigg\{\frac{1}{m_s^2}\left[\frac{31}{120}(1-x)x\pi^4-\frac{107}{36}(1-x)x\pi^2-\frac{51157}{648}(1-x)x+\frac{659}{18}(1-x)x\zeta_3+\frac{88}{3}(1-x)xL_s\right]$$
$$+\frac{1}{m_s}\Bigg[\frac{31}{120}\left(2x^2-2x+3\right)\pi^4-\frac{107}{36}\left(2x^2-2x+3\right)\pi^2-\frac{51157}{648}\left(2x^2-2x+3\right)+\frac{659}{18}\left(2x^2-2x+3\right)\zeta_3$$
$$+\frac{88}{3}\left(2x^2-2x+3\right)L_s\Bigg]+\Bigg[\frac{1}{60}\left(-684x^3+684x^2-52x-93\right)\pi^4+\frac{1}{18}\Big(1296x^3-1224x^2-346x$$
$$-\frac{88}{1-x}+465-\frac{108}{x}\Big)\pi^2+\frac{8}{3}\text{Li}_2(x)\pi^2+\frac{1}{6}\left(6x^3-6x^2+x\right)L_x^4+\frac{1}{6}\left(6x^3-6x^2-x+8\right)L_y^4$$
$$-\frac{2}{3}\left(36x^3-30x^2+11x-\frac{3}{1-x}+3\right)L_x^3-\frac{4}{3}\left(6x^3-6x^2+x\right)L_mL_x^3$$
$$-\frac{2}{9}\left(108x^3-126x^2+34x-56+\frac{15}{x}-\frac{9}{x^2}\right)L_y^3-\frac{4}{3}\left(6x^3-6x^2+x\right)L_mL_y^3+\frac{2}{3}\left(6x^3-6x^2+3x-8\right)L_xL_y^3$$
$$+\left(2\left(6x^3-6x^2+x\right)\pi^2+12x\right)L_m^2+2\left(6x^3-6x^2+x\right)L_m^2L_x^2+\Bigg(2\left(36x^3-10x^2-6x+\frac{1}{1-x}+2\right)$$
$$-\frac{7}{3}\left(6x^3-6x^2+x\right)\pi^2\Bigg)L_x^2-4\left(6x^3-6x^2+x\right)\text{Li}_2(x)L_x^2+2\left(36x^3-24x^2+7x-\frac{3}{1-x}+3\right)L_mL_x^2$$
$$+2\left(6x^3-6x^2+x\right)L_m^2L_y^2+\left(-6x^3+6x^2-5x+12\right)L_x^2L_y^2+\Bigg(\frac{1}{3}\left(-42x^3+42x^2-13x+20\right)\pi^2$$
$$+\frac{1}{9}\left(648x^3-1044x^2+491x+\frac{132}{1-x}-634+\frac{198}{x}-\frac{108}{x^2}\right)\Bigg)L_y^2+2\left(36x^3-48x^2+15x-5+\frac{5}{x}-\frac{3}{x^2}\right)L_mL_y^2$$
$$-\frac{22}{3}(x-2)L_sL_y^2+6\left(2x^2-3x+1\right)L_xL_y^2+\frac{1}{324}\Bigg(-110054x+\Big(-46656x^3+46656x^2+28008x+\frac{6480}{1-x}$$
$$-73764\Big)\zeta_3-\frac{15480}{1-x}+168951\Bigg)-4\left(36x^3-36x^2+24x+\frac{1}{1-x}\right)\text{Li}_3(x)-8\left(6x^3-6x^2+x\right)\text{Li}_4(x)$$
$$+\left(2\left(36x^3-36x^2+11x-1\right)\pi^2-\frac{82x}{9}+\left(-48x^3+48x^2-8x\right)\zeta_3+\frac{298}{9(1-x)}-\frac{944}{9}\right)L_m$$
$$-8\left(6x^3-6x^2+x\right)\text{Li}_3(x)L_m-\frac{22}{3}\left(-17x-\frac{2}{1-x}+26\right)L_s+\frac{44}{3}\left(x-\frac{1}{1-x}+2\right)L_mL_s$$
$$-8\left(3x^2-2x\right)L_m^2L_x+\left(\frac{2}{3}\left(42x^2-7x-\frac{11}{1-x}+17\right)\pi^2+12\left(-4x^2+2x-\frac{1}{1-x}+2\right)\right)L_x$$
$$+4\left(36x^3-36x^2+24x+\frac{1}{1-x}\right)\text{Li}_2(x)L_x+8\left(6x^3-6x^2+x\right)\text{Li}_3(x)L_x-4\left(24x^2-16x+\frac{2}{1-x}+1\right)L_mL_x$$
$$+8\left(6x^3-6x^2+x\right)\text{Li}_2(x)L_mL_x-\frac{2}{3}\left(6x^3-6x^2+x\right)L_x^3L_y+8\left(3x^2-2x\right)L_m^2L_y$$
$$-4\left(-18x^3+21x^2-13x-\frac{1}{1-x}+1\right)L_x^2L_y+4\left(6x^3-6x^2+x\right)L_mL_x^2L_y$$
$$+\left(48x^2-\frac{433x}{9}-\frac{2}{9}\left(126x^2-62x+28-\frac{45}{x}+\frac{27}{x^2}\right)\pi^2+(32-24x)\zeta_3-\frac{166}{9(1-x)}+\frac{382}{9}-\frac{12}{x}\right)L_y$$
$$+16(x-3)\text{Li}_3(x)L_y+\frac{4}{3}\left(72x^2-55x-\frac{11}{1-x}+37-\frac{9}{x}\right)L_mL_y-\frac{22}{3}\left(x-\frac{2}{1-x}+2\right)L_sL_y$$
$$-4\left(6x^3-6x^2+x\right)L_m^2L_xL_y+\left(\frac{2}{3}\left(42x^3-42x^2+7x+4\right)\pi^2-4\left(36x^3-34x^2+2x-\frac{2}{1-x}+1\right)\right)L_xL_y$$
$$-16(x-3)\text{Li}_2(x)L_xL_y-4\left(36x^3-36x^2+11x-1\right)L_mL_xL_y-4\left(-36x^3+36x^2-12x+\frac{1}{1-x}\right.$$

$$+\frac{5}{x}-\frac{3}{x^2}\Big)\,S_{1,2}(x)+8\left(6x^3-6x^2+x\right)L_m S_{1,2}(x)-16(x-3)L_x S_{1,2}(x)+16L_y S_{1,2}(x)$$

$$+16\left(3x^3-3x^2+x+1\right)S_{1,3}(x)+16(x-3)S_{2,2}(x)\big]\Big\}$$

$$+C_F{}^2\left\{\frac{1}{m_s{}^2}\left[-\frac{11}{45}(1-x)x\pi^4+\frac{29}{6}(1-x)x\pi^2+\frac{255}{8}(1-x)x-30(1-x)x\zeta_3\right]\right.$$

$$+\frac{1}{m_s}\left[-\frac{11}{45}\left(2x^2-2x+3\right)\pi^4+\frac{29}{6}\left(2x^2-2x+3\right)\pi^2+\frac{255}{8}\left(2x^2-2x+3\right)-30\left(2x^2-2x+3\right)\zeta_3\right]$$

$$+\left[\frac{1}{45}\left(1026x^3-1026x^2+149x+22\right)\pi^4+\frac{1}{3}\left(-432x^3+408x^2+76x+\frac{72}{1-x}-161+\frac{24}{x}\right)\pi^2-\frac{8}{3}x\mathrm{Li}_2(x)\pi^2\right.$$

$$+\frac{1}{3}\left(-6x^3+6x^2-x\right)L_x^4-\frac{2}{3}\left(3x^3-3x^2-2x+7\right)L_y^4+\frac{4}{3}\left(36x^3-30x^2+11x-\frac{3}{1-x}+3\right)L_x^3$$

$$+\frac{8}{3}\left(6x^3-6x^2+x\right)L_m L_x^3-\frac{2}{3}\left(-72x^3+84x^2-11x+\frac{6}{1-x}+2-\frac{12}{x}+\frac{4}{x^2}\right)L_y^3+\frac{8}{3}\left(6x^3-6x^2+x\right)L_m L_y^3$$

$$-\frac{4}{3}\left(6x^3-6x^2+3x-8\right)L_x L_y^3+\left(-4\left(6x^3-6x^2+x\right)\pi^2-2\left(12x-\frac{1}{1-x}+1\right)\right)L_m^2$$

$$-4\left(6x^3-6x^2+x\right)L_m^2 L_x^2+\left(\frac{14}{3}\left(6x^3-6x^2+x\right)\pi^2-4\left(36x^3-10x^2-6x+\frac{1}{1-x}+2\right)\right)L_x^2$$

$$+8\left(6x^3-6x^2+x\right)\mathrm{Li}_2(x)L_x^2-4\left(36x^3-24x^2+7x-\frac{3}{1-x}+3\right)L_m L_x^2-4\left(6x^3-6x^2+x\right)L_m^2 L_y^2$$

$$+2\left(6x^3-6x^2+5x-12\right)L_x^2 L_y^2+\left(-144x^3+232x^2-83x+\frac{2}{3}\left(42x^3-42x^2+5x-4\right)\pi^2-\frac{14}{1-x}\right.$$

$$\left.+46-\frac{24}{x}+\frac{16}{x^2}\right)L_y^2+4\left(-36x^3+48x^2-16x+\frac{1}{1-x}+3-\frac{6}{x}+\frac{2}{x^2}\right)L_m L_y^2-12\left(2x^2-3x+1\right)L_x L_y^2$$

$$+\frac{1}{4}\left(586x+\left(1152x^3-1152x^2-48x-\frac{64}{1-x}+688\right)\zeta_3+\frac{144}{1-x}-909\right)$$

$$+8\left(36x^3-36x^2+24x+\frac{1}{1-x}\right)\mathrm{Li}_3(x)+16\left(6x^3-6x^2+x\right)\mathrm{Li}_4(x)+\left(-4\left(36x^3-36x^2+11x-1\right)\pi^2\right.$$

$$\left.-18x+\left(96x^3-96x^2+16x\right)\zeta_3-\frac{26}{1-x}+96\right)L_m+16\left(6x^3-6x^2+x\right)\mathrm{Li}_3(x)L_m+16\left(3x^2-2x\right)L_m^2 L_x$$

$$+\left(-\frac{4}{3}\left(42x^2-7x-\frac{11}{1-x}+17\right)\pi^2-24\left(-4x^2+2x-\frac{1}{1-x}+2\right)\right)L_x$$

$$-8\left(36x^3-36x^2+24x+\frac{1}{1-x}\right)\mathrm{Li}_2(x)L_x-16\left(6x^3-6x^2+x\right)\mathrm{Li}_3(x)L_x$$

$$+8\left(24x^2-16x+\frac{2}{1-x}+1\right)L_m L_x-16\left(6x^3-6x^2+x\right)\mathrm{Li}_2(x)L_m L_x+\frac{4}{3}\left(6x^3-6x^2+x\right)L_x^3 L_y$$

$$-16\left(3x^2-2x\right)L_m^2 L_y+8\left(-18x^3+21x^2-13x-\frac{1}{1-x}+1\right)L_x^2 L_y-8\left(6x^3-6x^2+x\right)L_m L_x^2 L_y$$

$$+\left(-96x^2+127x+\frac{8}{3}\left(21x^2-16x+\frac{3}{1-x}+11-\frac{9}{x}+\frac{3}{x^2}\right)\pi^2+(24x-16)\zeta_3+\frac{14}{1-x}-50+\frac{16}{x}\right)L_y$$

$$-32(x-3)\mathrm{Li}_3(x)L_y-4\left(48x^2-41x-\frac{2}{1-x}+15-\frac{4}{x}\right)L_m L_y+8\left(6x^3-6x^2+x\right)L_m^2 L_x L_y$$

$$+\left(8\left(36x^3-34x^2+2x-\frac{2}{1-x}+1\right)-\frac{4}{3}\left(42x^3-42x^2+7x+4\right)\pi^2\right)L_x L_y+32(x-3)\mathrm{Li}_2(x)L_x L_y$$

$$+8\left(36x^3-36x^2+11x-1\right)L_m L_x L_y-4\left(72x^3-72x^2+23x-\frac{4}{1-x}+10-\frac{12}{x}+\frac{4}{x^2}\right)S_{1,2}(x)$$

$$-16\left(6x^3-6x^2+x\right)L_m S_{1,2}(x)+32(x-3)L_x S_{1,2}(x)+8(x-6)L_y S_{1,2}(x)-8\left(12x^3-12x^2+x+10\right)S_{1,3}(x)$$

$$-16(3x-8)S_{2,2}(x)\big]\Big\}$$

$$+n_f T_F C_F\left\{\frac{1}{m_s{}^2}\left[\frac{7}{9}(1-x)x\pi^2+\frac{4085}{162}(1-x)x-\frac{2}{9}(1-x)x\zeta_3-\frac{32}{3}(1-x)xL_s\right]\right.$$

$$+\frac{1}{m_s}\left[\frac{7}{9}\left(2x^2-2x+3\right)\pi^2+\frac{4085}{162}\left(2x^2-2x+3\right)-\frac{2}{9}\left(2x^2-2x+3\right)\zeta_3-\frac{32}{3}\left(2x^2-2x+3\right)\mathrm{L}_s\right]$$

$$+\left[\frac{44}{45}(x+1)\pi^4-\frac{2}{9}\left(-26x+\frac{58}{1-x}+27-\frac{54}{x}\right)\pi^2-16\mathrm{Li}_2(x)\pi^2+\frac{4}{3}\left(2x^2-2x-\frac{4}{1-x}+\frac{3}{(x-1)^2}+1\right)\mathrm{L}_x^3\right.$$

$$-\frac{4}{9}\left(6x^2-10x+11-\frac{12}{x}+\frac{9}{x^2}\right)\mathrm{L}_y^3-4\left(10x-\frac{16}{1-x}+\frac{6}{(x-1)^2}+7\right)\mathrm{L}_x^2-4\left(\frac{3}{(x-1)^2}+1-\frac{4}{1-x}\right)\mathrm{L}_m\mathrm{L}_x^2$$

$$-8(x-2)\mathrm{L}_x^2\mathrm{L}_y^2+\frac{4}{9}\left(-103x-\frac{12}{1-x}+203-\frac{144}{x}+\frac{54}{x^2}\right)\mathrm{L}_y^2+4\left(1-\frac{4}{x}+\frac{3}{x^2}\right)\mathrm{L}_m\mathrm{L}_y^2+\frac{8}{3}(x-2)\mathrm{L}_s\mathrm{L}_y^2$$

$$-4(2x-1)\mathrm{L}_x\mathrm{L}_y^2+\frac{1}{81}\left(8854x+\left(-1296x^2+10296x-\frac{2592}{1-x}+\frac{1944}{(x-1)^2}-1188\right)\zeta_3+\frac{1368}{1-x}-13623\right)$$

$$+8\left(2x^2-12x+\frac{4}{1-x}-\frac{3}{(x-1)^2}\right)\mathrm{Li}_3(x)+\left(\frac{8}{9}\left(13x-\frac{13}{1-x}+32\right)-4(2x-1)\pi^2\right)\mathrm{L}_m$$

$$+\frac{8}{3}\left(-17x-\frac{2}{1-x}+26\right)\mathrm{L}_s-\frac{16}{3}\left(x-\frac{1}{1-x}+2\right)\mathrm{L}_m\mathrm{L}_s-16\left(x^2-x\right)\mathrm{L}_m^2\mathrm{L}_x+\left(24\left(-4x^2+2x-\frac{1}{1-x}+2\right)\right.$$

$$\left.-\frac{4}{3}\left(4x^2-8x-\frac{12}{1-x}+\frac{9}{(x-1)^2}+9\right)\pi^2\right)\mathrm{L}_x-8\left(2x^2-12x+\frac{4}{1-x}-\frac{3}{(x-1)^2}\right)\mathrm{Li}_2(x)\mathrm{L}_x$$

$$-8\left(16x^2-14x+\frac{3}{1-x}\right)\mathrm{L}_m\mathrm{L}_x+16\left(x^2-x\right)\mathrm{L}_m^2\mathrm{L}_y+4\left(-2x^2+10x-\frac{4}{1-x}+\frac{3}{(x-1)^2}+1\right)\mathrm{L}_x^2\mathrm{L}_y$$

$$+\left(96x^2-\frac{1372x}{9}+\frac{4}{9}\left(12x^2+4x+7-\frac{36}{x}+\frac{27}{x^2}\right)\pi^2+(64-32x)\zeta_3+\frac{56}{9(1-x)}-\frac{56}{9}+\frac{24}{x}\right)\mathrm{L}_y$$

$$+32(x-2)\mathrm{Li}_3(x)\mathrm{L}_y+\frac{8}{3}\left(48x^2-56x+\frac{2}{1-x}+2+\frac{9}{x}\right)\mathrm{L}_m\mathrm{L}_y+\frac{8}{3}\left(x-\frac{2}{1-x}+2\right)\mathrm{L}_s\mathrm{L}_y$$

$$+\left(\frac{16}{3}(x-2)\pi^2+8(2x-1)\right)\mathrm{L}_x\mathrm{L}_y-32(x-2)\mathrm{Li}_2(x)\mathrm{L}_x\mathrm{L}_y+8(2x-1)\mathrm{L}_m\mathrm{L}_x\mathrm{L}_y$$

$$\left.+8\left(2x^2+8x-10+\frac{4}{x}-\frac{3}{x^2}\right)\mathrm{S}_{1,2}(x)-32(x-2)\mathrm{L}_x\mathrm{S}_{1,2}(x)-96\mathrm{S}_{2,2}(x)\right]\right\}\,,\tag{4.12}$$

$$\mathcal{J}_3^{(0\times2)}(m_s,x)=$$

$$C_AC_F\left\{\frac{1}{m_s^2}\left[\frac{31}{240}(1-x)x\pi^4-\frac{107}{72}(1-x)x\pi^2-\frac{51157(1-x)x}{1296}+\frac{659}{36}(1-x)x\zeta_3+\frac{44}{3}(1-x)x\mathrm{L}_s\right]\right.$$

$$+\frac{1}{m_s}\left[\frac{31}{240}\left(4x^2-4x+3\right)\pi^4-\frac{107}{72}\left(4x^2-4x+3\right)\pi^2-\frac{51157\left(4x^2-4x+3\right)}{1296}+\frac{659}{36}\left(4x^2-4x+3\right)\zeta_3\right.$$

$$\left.+\frac{44}{3}\left(4x^2-4x+3\right)\mathrm{L}_s\right]+\left[-\frac{31}{20}\left(x^2-x+1\right)\pi^4+\frac{107}{6}\left(x^2-x+1\right)\pi^2+\frac{1}{108}\left(51157x^2-51157x\right.\right.$$

$$\left.\left.+\left(-23724x^2+23724x-23724\right)\zeta_3+51157\right)-176\left(x^2-x+1\right)\mathrm{L}_s\right]\right\}$$

$$+C_F^2\left\{\frac{1}{m_s^2}\left[-\frac{11}{90}(1-x)x\pi^4+\frac{29}{12}(1-x)x\pi^2+\frac{255}{16}(1-x)x-15(1-x)x\zeta_3\right]\right.$$

$$+\frac{1}{m_s}\left[-\frac{11}{90}\left(4x^2-4x+3\right)\pi^4+\frac{29}{12}\left(4x^2-4x+3\right)\pi^2+\frac{255}{16}\left(4x^2-4x+3\right)-15\left(4x^2-4x+3\right)\zeta_3\right]$$

$$\left.+\left[\frac{22}{15}\left(x^2-x+1\right)\pi^4-29\left(x^2-x+1\right)\pi^2+\frac{45}{4}\left(-17x^2+17x+\left(16x^2-16x+16\right)\zeta_3-17\right)\right]\right\}$$

$$+n_fT_FC_F\left\{\frac{1}{m_s^2}\left[\frac{7}{18}(1-x)x\pi^2+\frac{4085}{324}(1-x)x-\frac{1}{9}(1-x)x\zeta_3-\frac{16}{3}(1-x)x\mathrm{L}_s\right]\right.$$

$$+\frac{1}{m_s}\left[\frac{7}{18}\left(4x^2-4x+3\right)\pi^2+\frac{4085}{324}\left(4x^2-4x+3\right)+\frac{1}{9}\left(-4x^2+4x-3\right)\zeta_3-\frac{16}{3}\left(4x^2-4x+3\right)\mathrm{L}_s\right]$$

$$\left.+\left[-\frac{14}{3}\left(x^2-x+1\right)\pi^2+\frac{1}{27}\left(-4085x^2+4085x+\left(36x^2-36x+36\right)\zeta_3-4085\right)+64\left(x^2-x+1\right)\mathrm{L}_s\right]\right\}\,,\tag{4.13}$$

$$\mathcal{J}_4^{(0\times2)}(m_s,x) =$$

$$n_f T_F C_F \left\{ \left[\frac{22}{45}(x+1)\pi^4 - \frac{2}{3}\left(-2x + \frac{9}{1-x} + 1 - \frac{9}{x}\right)\pi^2 - 8\mathrm{Li}_2(x)\pi^2 - 2(2x-1)\mathrm{L}_m\pi^2 \right. \right.$$

$$+ \frac{2}{3}\left(2x^2 - 2x - \frac{4}{1-x} + \frac{3}{(x-1)^2} + 1\right)\mathrm{L}_x^3 - \frac{2}{3}\left(2x^2 - 2x + 1 - \frac{4}{x} + \frac{3}{x^2}\right)\mathrm{L}_y^3$$

$$- 2\left(10x - \frac{16}{1-x} + \frac{6}{(x-1)^2} + 7\right)\mathrm{L}_x^2 - 2\left(\frac{3}{(x-1)^2} + 1 - \frac{4}{1-x}\right)\mathrm{L}_m\mathrm{L}_x^2 - 4(x-2)\mathrm{L}_x^2\mathrm{L}_y^2$$

$$+ 2\left(-10x + 17 - \frac{16}{x} + \frac{6}{x^2}\right)\mathrm{L}_y^2 + 2\left(1 - \frac{4}{x} + \frac{3}{x^2}\right)\mathrm{L}_m\mathrm{L}_y^2 - 2(2x-1)\mathrm{L}_x\mathrm{L}_y^2$$

$$- 4\left(2x^2 - 16x + \frac{4}{1-x} - \frac{3}{(x-1)^2} + 2\right)\zeta_3 + 4\left(2x^2 - 12x + \frac{4}{1-x} - \frac{3}{(x-1)^2}\right)\mathrm{Li}_3(x)$$

$$- 8\left(x^2 - x\right)\mathrm{L}_m^2\mathrm{L}_x + \left(12\left(-4x^2 + 2x - \frac{1}{1-x} + 2\right) - \frac{2}{3}\left(4x^2 - 8x - \frac{12}{1-x} + \frac{9}{(x-1)^2} + 9\right)\pi^2\right)\mathrm{L}_x$$

$$- 4\left(2x^2 - 12x + \frac{4}{1-x} - \frac{3}{(x-1)^2}\right)\mathrm{Li}_2(x)\mathrm{L}_x - 4\left(16x^2 - 14x + \frac{3}{1-x}\right)\mathrm{L}_m\mathrm{L}_x + 8\left(x^2 - x\right)\mathrm{L}_m^2\mathrm{L}_y$$

$$+ 2\left(-2x^2 + 10x - \frac{4}{1-x} + \frac{3}{(x-1)^2} + 1\right)\mathrm{L}_x^2\mathrm{L}_y + \left(48x^2 - 72x + \frac{2}{3}\left(4x^2 + 5 - \frac{12}{x} + \frac{9}{x^2}\right)\pi^2\right.$$

$$\left. + (32 - 16x)\zeta_3 + \frac{12}{x}\right)\mathrm{L}_y + 16(x-2)\mathrm{Li}_3(x)\mathrm{L}_y + 4\left(16x^2 - 18x + 2 + \frac{3}{x}\right)\mathrm{L}_m\mathrm{L}_y$$

$$+ \left(\frac{8}{3}(x-2)\pi^2 + 4(2x-1)\right)\mathrm{L}_x\mathrm{L}_y - 16(x-2)\mathrm{Li}_2(x)\mathrm{L}_x\mathrm{L}_y + 4(2x-1)\mathrm{L}_m\mathrm{L}_x\mathrm{L}_y$$

$$\left. \left. + 4\left(2x^2 + 8x - 10 + \frac{4}{x} - \frac{3}{x^2}\right)\mathrm{S}_{1,2}(x) - 16(x-2)\mathrm{L}_x\mathrm{S}_{1,2}(x) - 48\mathrm{S}_{2,2}(x) \right] \right\} , \tag{4.14}$$

where L_m, L_s, L_x and L_y are defined as

$$\mathrm{L}_m = \log(m_s) , \quad \mathrm{L}_s = \log\left(\frac{s}{\mu^2}\right) , \quad \mathrm{L}_x = \log(x) , \quad \mathrm{L}_y = \log(1-x) . \tag{4.15}$$

4.2 One-loop squared contribution

In this section, we give explicit expressions for the finite remainder of the one-loop squared contribution $\mathcal{F}_{inite,}^{(1\times1)}$ defined as

$$\mathcal{F}_{inite,}^{(1\times1)}(s,t,u,m,\mu) = \mathcal{A}^{\mathrm{NNLO}\,(1\times1)}(s,t,u,m,\mu) - \mathcal{C}_{\mathrm{atani}}^{(1\times1)}(s,t,u,m,\mu) , \tag{4.16}$$

The EW structure of the finite remainder for the one-loop squared corrections, similarly to the case of the two-loop corrections, can be factorised as

$$\mathcal{F}_{inite,\,down}^{(1\times1)} = N\,C_F^{\,2}\sum_{i=1,3} c_i \mathcal{J}_{i,\,down}^{(1\times1)}\left(m_s, x, \frac{s}{\mu^2}\right) . \tag{4.17}$$

Our result then reads:

$$\mathcal{J}_1^{(1\times1)} = \frac{64(1-x)x}{m_s^2} + \frac{192}{m_s} + \left[-4\left(-\frac{2}{x} - \frac{1}{x^2} - \frac{1}{x^3} + 1 - \frac{1}{1-x}\right)\mathrm{L}_y^4 + 8\left(\frac{3}{x} + \frac{2}{x^2} + \frac{3}{1-x}\right)\mathrm{L}_y^3 \right.$$

$$+ \left(4\left(\frac{6}{x} + 15 - \frac{5}{1-x}\right) - 16\left(-\frac{2}{x} - \frac{1}{x^2} - \frac{1}{x^3} + 1 - \frac{1}{1-x}\right)\pi^2\right)\mathrm{L}_y^2$$

$$+ \left(16\left(\frac{3}{x} + \frac{2}{x^2} + \frac{3}{1-x}\right)\pi^2 + 8\left(7 - \frac{5}{1-x}\right)\right)\mathrm{L}_y - 4\left(-\frac{4}{x} + 1 - \frac{9}{1-x}\right)\pi^2$$

$$-4\left(-32x-\frac{83}{1-x}+83\right)+128\left(x-\frac{1}{1-x}+2\right)\mathrm{L}_m\bigg],$$

$$\mathcal{J}_2{}^{(1\times1)}=\frac{64(1-x)x}{m_s{}^2}+\frac{1}{m_s}\left[64\left(2x^2-2x+3\right)\right]+\left[-32(x-2)\mathrm{L}_y^2-32\left(x-\frac{2}{1-x}+2\right)\mathrm{L}_y\right.$$

$$\left.-32\left(-9x-\frac{2}{1-x}+14\right)+64\left(x-\frac{1}{1-x}+2\right)\mathrm{L}_m\right],$$

$$\mathcal{J}_3{}^{(1\times1)}=\frac{32(1-x)x}{m_s{}^2}+\frac{1}{m_s}\left[32\left(4x^2-4x+3\right)\right]+\left[-384\left(x^2-x+1\right)\right]. \tag{4.18}$$

5. Conclusions

In this work we have calculated the NNLO QCD virtual corrections for the process $q\bar{q}\rightarrow W^+W^-$ in the limit of small vector boson mass. The $\overline{\mathrm{MS}}$ renormalised amplitude is still infrared divergent and contains poles up to $\mathcal{O}(1/\epsilon^4)$. We checked that the infrared structure of our result agrees with the prediction of Catani's formalism for the infrared structure of QCD amplitudes.

The main result of our paper has been given as the finite remainder of the NNLO two-loop and one-loop virtual corrections after subtraction of the structure predicted by Catani's formalism. This is a first step towards the complete evaluation of the virtual corrections. In a forthcoming publication, we will derive a series expansion in the mass and integrate the result numerically. This will require the present result as a starting point.

To complete the NNLO project one still needs to consider $2\rightarrow3$ real-virtual contributions and $2\rightarrow4$ real ones. The real-virtual corrections are known from the NLO studies on $WW+jet$ production in refs. [46, 47]. The integration over the full phase space would require additional subtraction terms, similar to those constructed in ref. [48].

Acknowledgments

A. $\langle\mathcal{M}^{(0)}|\mathcal{M}^{(1)}_{\mathrm{finite}}\rangle$ to order ϵ^2

Here we present the expression for the one-loop result, $\langle\mathcal{M}^{(0)}|\mathcal{M}^{(1)}_{\mathrm{finite}}\rangle$ up to order ϵ^2 for down-type quarks. This result completes the list of the elements needed in eq. (2.8) in order to have the perturbative expansion of the amplitude up to order α_s^2 in the high energy limit.

$$\langle\mathcal{M}^{(0)}|\mathcal{M}^{(1)}_{\mathrm{finite}}\rangle=NC_F\sum_{i=1,3}c_i\mathcal{J}_i^{(0\times1)}. \tag{A.1}$$

$$\mathcal{J}_1^{(0\times1)}=\left\{\frac{1}{m_s^2}\left[-16(1-x)x\right]+\frac{1}{m_s}\left[-48\right]+\left[-8\left(1-\frac{1}{1-x}\right)\mathrm{L}_y^2-8\left(1-\frac{1}{1-x}\right)\mathrm{L}_y\right.\right.$$

$$\left.\left.+8\left(-2x-\frac{9}{1-x}+9\right)-16\left(x-\frac{1}{1-x}+2\right)\mathrm{L}_m\right]\right\}$$

$$+ i\pi \left\{ -16\mathrm{L}_y \left(1 - \frac{1}{1-x} \right) - 8 \left(1 - \frac{1}{1-x} \right) \right\}$$

$$+ \epsilon \left\{ \frac{1}{m_s^2} \left[-32(1-x)x + 8(1-x)\zeta_3 x + 16(1-x)\mathrm{L}_s x \right] + \frac{1}{m_s} \left[24\zeta_3 + 48\mathrm{L}_s - 32 \right] \right.$$

$$+ \left[\frac{16}{3} \left(1 - \frac{1}{1-x} \right) \mathrm{L}_y^3 + 4 \left(3 - \frac{5}{1-x} \right) \mathrm{L}_y^2 + 8 \left(1 - \frac{1}{1-x} \right) \mathrm{L}_s \mathrm{L}_y^2 + \right.$$

$$\left(8 \left(1 - \frac{1}{1-x} \right) \pi^2 - 8 \left(1 + \frac{1}{1-x} \right) \right) \mathrm{L}_y + 8 \left(1 - \frac{1}{1-x} \right) \mathrm{L}_s \mathrm{L}_y + 4 \left(1 - \frac{1}{1-x} \right) \pi^2$$

$$+ 8 \left(x - \frac{1}{1-x} + 2 \right) \mathrm{L}_m^2 - 32 \left(x + \left(1 - \frac{1}{1-x} \right) \zeta_3 \right) - 16 \left(x - \frac{2}{1-x} + 4 \right) \mathrm{L}_m$$

$$\left. \left. - 8 \left(-2x - \frac{9}{1-x} + 9 \right) \mathrm{L}_s + 16 \left(x - \frac{1}{1-x} + 2 \right) \mathrm{L}_m \mathrm{L}_s - 16 \left(1 - \frac{1}{1-x} \right) \mathrm{S}_{1,2}(x) \right] \right\}$$

$$+ \epsilon \, i\pi \left\{ \frac{1}{m_s^2} \left[-16(1-x)x \right] + \frac{1}{m_s} \left[-48 \right] + \left[8 \left(1 - \frac{1}{1-x} \right) \mathrm{L}_y^2 + 16 \left(1 - \frac{2}{1-x} \right) \mathrm{L}_y \right. \right.$$

$$+ 16 \left(1 - \frac{1}{1-x} \right) \mathrm{L}_s \mathrm{L}_y + 16 \left(-x - \frac{5}{1-x} + 4 \right) + 16 \left(1 - \frac{1}{1-x} \right) \mathrm{Li}_2(x)$$

$$\left. \left. - 16 \left(x - \frac{1}{1-x} + 2 \right) \mathrm{L}_m + 8 \left(1 - \frac{1}{1-x} \right) \mathrm{L}_s \right] \right\}$$

$$+ \epsilon^2 \left\{ \frac{1}{m_s^2} \left[\frac{2}{15}(1-x)x\pi^4 + \frac{28}{3}(1-x)x\pi^2 - 8(1-x)x\mathrm{L}_s^2 - 64(1-x)x + 32(1-x)x\mathrm{L}_s \right. \right.$$

$$\left. + \zeta_3(12(1-x)x - 8(1-x)x\mathrm{L}_s) \right] + \frac{1}{m_s} \left[\frac{2\pi^4}{5} + 28\pi^2 - 24\mathrm{L}_s^2 + \zeta_3(4 - 24\mathrm{L}_s) + 32\mathrm{L}_s - 64 \right]$$

$$+ \left[-\frac{8}{15} \left(1 - \frac{1}{1-x} \right) \pi^4 - \frac{2}{3} \left(-14x - \frac{69}{1-x} + 57 \right) \pi^2 - 8 \left(1 - \frac{1}{1-x} \right) \mathrm{Li}_2(x)\pi^2 \right.$$

$$- 2 \left(1 - \frac{1}{1-x} \right) \mathrm{L}_y^4 - \frac{8}{3} \left(x - \frac{1}{1-x} + 2 \right) \mathrm{L}_m^3 - \frac{4}{3} \left(5 - \frac{9}{1-x} \right) \mathrm{L}_y^3 - \frac{16}{3} \left(1 - \frac{1}{1-x} \right) \mathrm{L}_s \mathrm{L}_y^3$$

$$+ 8 \left(x - \frac{2}{1-x} + 4 \right) \mathrm{L}_m^2 + 4 \left(-2x - \frac{9}{1-x} + 9 \right) \mathrm{L}_s^2 - 8 \left(x - \frac{1}{1-x} + 2 \right) \mathrm{L}_m \mathrm{L}_s^2$$

$$- 4 \left(1 - \frac{1}{1-x} \right) \mathrm{L}_s^2 \mathrm{L}_y^2 + \left(12 \left(1 + \frac{1}{1-x} \right) - \frac{10}{3} \left(1 - \frac{1}{1-x} \right) \pi^2 \right) \mathrm{L}_y^2 - 4 \left(3 - \frac{5}{1-x} \right) \mathrm{L}_s \mathrm{L}_y^2$$

$$+ 8 \left(-8x + \left(2 - \frac{2}{1-x} \right) \zeta_3 - \frac{9}{1-x} + 9 \right) + \left(\frac{28}{3} \left(x - \frac{1}{1-x} + 2 \right) \pi^2 - 16 \left(2x - \frac{3}{1-x} + 6 \right) \right) \mathrm{L}_m$$

$$- 8 \left(x - \frac{1}{1-x} + 2 \right) \mathrm{L}_m^2 \mathrm{L}_s + \left(-4 \left(1 - \frac{1}{1-x} \right) \pi^2 + 32x + \left(32 - \frac{32}{1-x} \right) \zeta_3 \right) \mathrm{L}_s$$

$$+ 16 \left(x - \frac{2}{1-x} + 4 \right) \mathrm{L}_m \mathrm{L}_s - 4 \left(1 - \frac{1}{1-x} \right) \mathrm{L}_s^2 \mathrm{L}_y + \left(-\frac{2}{3} \left(11 - \frac{23}{1-x} \right) \pi^2 - 24 \left(1 + \frac{1}{1-x} \right) \right) \mathrm{L}_y$$

$$+ \left(8 \left(1 + \frac{1}{1-x} \right) - 8 \left(1 - \frac{1}{1-x} \right) \pi^2 \right) \mathrm{L}_s \mathrm{L}_y + 16 \left(1 - \frac{2}{1-x} \right) \mathrm{S}_{1,2}(x) + 16 \left(1 - \frac{1}{1-x} \right) \mathrm{L}_s \mathrm{S}_{1,2}(x)$$

$$\left. \left. + 16 \left(1 - \frac{1}{1-x} \right) \mathrm{L}_y \mathrm{S}_{1,2}(x) + 16 \left(1 - \frac{1}{1-x} \right) \mathrm{S}_{1,3}(x) - 16 \left(1 - \frac{1}{1-x} \right) \mathrm{S}_{2,2}(x) \right] \right\}$$

$$+ \epsilon^2 \, i\pi \left\{ \frac{1}{m_s^2} \left[-32(1-x)x + 8(1-x)\zeta_3 x + 16(1-x)\mathrm{L}_s x \right] + \frac{1}{m_s} \left[24\zeta_3 + 48\mathrm{L}_s - 32 \right] \right.$$

$$+ \left[-\frac{8}{3} \left(1 - \frac{1}{1-x} \right) \mathrm{L}_y^3 - 8 \left(1 - \frac{2}{1-x} \right) \mathrm{L}_y^2 - 8 \left(1 - \frac{1}{1-x} \right) \mathrm{L}_s \mathrm{L}_y^2 - 8 \left(1 - \frac{1}{1-x} \right) \mathrm{L}_s^2 \mathrm{L}_y \right.$$

$$+ \left(4 \left(1 - \frac{1}{1-x} \right) \pi^2 + 16 \left(1 + \frac{1}{1-x} \right) \right) \mathrm{L}_y - 16 \left(1 - \frac{1}{1-x} \right) \mathrm{Li}_2(x)\mathrm{L}_y - 16 \left(1 - \frac{2}{1-x} \right) \mathrm{L}_s \mathrm{L}_y$$

$$+2\left(1-\frac{1}{1-x}\right)\pi^2+8\left(x-\frac{1}{1-x}+2\right)L_m^2-4\left(1-\frac{1}{1-x}\right)L_s^2$$

$$-8\left(4x+\left(4-\frac{4}{1-x}\right)\zeta_3+\frac{3}{1-x}+3\right)-16\left(1-\frac{2}{1-x}\right)\mathrm{Li}_2(x)+16\left(1-\frac{1}{1-x}\right)\mathrm{Li}_3(x)$$

$$-16\left(x-\frac{2}{1-x}+4\right)L_m-16\left(-x-\frac{5}{1-x}+4\right)L_s-16\left(1-\frac{1}{1-x}\right)\mathrm{Li}_2(x)L_s$$

$$+16\left(x-\frac{1}{1-x}+2\right)L_mL_s-16\left(1-\frac{1}{1-x}\right)S_{1,2}(x)\Big]\Big\}\,,\tag{A.2}$$

$$\mathcal{J}_2^{(0\times1)}=\Big\{\frac{1}{m_s^2}\left[-16(1-x)x\right]+\frac{1}{m_s}\left[-16\left(2x^2-2x+3\right)\right]+\Big[4(x-2)L_y^2+4\left(x-\frac{2}{1-x}+2\right)L_y$$

$$+4\left(-17x-\frac{2}{1-x}+26\right)-8\left(x-\frac{1}{1-x}+2\right)L_m\Big]\Big\}$$

$$+i\pi\Big\{\Big[4\left(x-\frac{2}{1-x}+2\right)+8(x-2)L_y\Big]\Big\}$$

$$+\epsilon\Big\{\frac{1}{m_s^2}\left[-32(1-x)x+8(1-x)\zeta_3x+16(1-x)L_sx\right]+\frac{1}{m_s}\left[-32\left(2x^2-2x+1\right)\right.$$

$$+8\left(2x^2-2x+3\right)\zeta_3+16\left(2x^2-2x+3\right)L_s\Big]+\Big[-\frac{8}{3}(x-2)L_y^3+2\left(\frac{2}{1-x}-5x\right)L_y^2$$

$$-4(x-2)L_sL_y^2+\left(-4(x-2)\pi^2-8\left(\frac{1}{1-x}-2x\right)\right)L_y-4\left(x-\frac{2}{1-x}+2\right)L_sL_y$$

$$-2\left(x-\frac{2}{1-x}+2\right)\pi^2+4\left(x-\frac{1}{1-x}+2\right)L_m^2+4\left(-17x+(8x-12)\zeta_3-\frac{2}{1-x}+18\right)$$

$$-8\left(x-\frac{2}{1-x}+4\right)L_m-4\left(-17x-\frac{2}{1-x}+26\right)L_s+8\left(x-\frac{1}{1-x}+2\right)L_mL_s+8(x-2)S_{1,2}(x)\Big]\Big\}$$

$$+\epsilon\,i\pi\Big\{\frac{1}{m_s^2}\left[-16(1-x)x\right]+\frac{1}{m_s}\left[-16\left(2x^2-2x+3\right)\right]+\Big[-4(x-2)L_y^2-8(2x-1)L_y$$

$$-8(x-2)L_sL_y+4\left(-13x-\frac{4}{1-x}+26\right)-8(x-2)\mathrm{Li}_2(x)-8\left(x-\frac{1}{1-x}+2\right)L_m$$

$$-4\left(x-\frac{2}{1-x}+2\right)L_s\Big]\Big\}$$

$$+\epsilon^2\Big\{\frac{1}{m_s^2}\Big[\frac{2}{15}(1-x)x\pi^4+\frac{28}{3}(1-x)x\pi^2-8(1-x)xL_s^2-64(1-x)x+32(1-x)xL_s$$

$$+\zeta_3(12(1-x)x-8(1-x)xL_s)\Big]+\frac{1}{m_s}\Big[\frac{2}{15}\left(2x^2-2x+3\right)\pi^4+\frac{28}{3}\left(2x^2-2x+3\right)\pi^2$$

$$-8\left(2x^2-2x+3\right)L_s^2-64\left(2x^2-2x+1\right)+32\left(2x^2-2x+1\right)L_s+\zeta_3\left(4\left(6x^2-6x+1\right)\right.$$

$$-8\left(2x^2-2x+3\right)L_s\Big)\Big]+\Big[\frac{4}{15}(2x-3)\pi^4+\frac{1}{3}\left(95x+\frac{26}{1-x}-182\right)\pi^2$$

$$+4(x-2)\mathrm{Li}_2(x)\pi^2+(x-2)L_y^4-\frac{4}{3}\left(x-\frac{1}{1-x}+2\right)L_m^3-\frac{2}{3}\left(-9x+\frac{2}{1-x}+2\right)L_y^3$$

$$+\frac{8}{3}(x-2)L_sL_y^3+4\left(x-\frac{2}{1-x}+4\right)L_m^2+2\left(-17x-\frac{2}{1-x}+26\right)L_s^2$$

$$-4\left(x-\frac{1}{1-x}+2\right)L_mL_s^2+2(x-2)L_s^2L_y^2+\left(\frac{5}{3}(x-2)\pi^2-4\left(x-\frac{1}{1-x}+1\right)\right)L_y^2$$

$$-2\left(\frac{2}{1-x}-5x\right)L_sL_y^2+8\left(-17x+(2x-1)\zeta_3-\frac{2}{1-x}+18\right)+\left(\frac{14}{3}\left(x-\frac{1}{1-x}+2\right)\pi^2\right.$$

$$-8\left(2x-\frac{3}{1-x}+6\right)\right)L_m - 4\left(x-\frac{1}{1-x}+2\right)L_m^2 L_s + \left(2\left(x-\frac{2}{1-x}+2\right)\pi^2 + 68x\right.$$

$$+(48-32x)\zeta_3 + \frac{8}{1-x}-72\right)L_s + 8\left(x-\frac{2}{1-x}+4\right)L_m L_s + 2\left(x-\frac{2}{1-x}+2\right)L_s^2 L_y$$

$$+\left(\frac{1}{3}\left(23x+\frac{2}{1-x}-14\right)\pi^2 - 4\left(-5x+\frac{4}{1-x}+2\right)\right)L_y$$

$$+\left(4(x-2)\pi^2 + 8\left(\frac{1}{1-x}-2x\right)\right)L_s L_y - 8(2x-1)S_{1,2}(x) - 8(x-2)L_s S_{1,2}(x)$$

$$-8(x-2)L_y S_{1,2}(x) - 8(x-2)S_{1,3}(x) + 8(x-2)S_{2,2}(x)]\}$$

$$+\epsilon^2 i\pi\left\{\frac{1}{m_s^2}\left[-32(1-x)x + 8(1-x)\zeta_3 x + 16(1-x)L_s x\right] + \frac{1}{m_s}\left[-32\left(2x^2-2x+1\right)\right.\right.$$

$$+8\left(2x^2-2x+3\right)\zeta_3 + 16\left(2x^2-2x+3\right)L_s\right] + \left[\frac{4}{3}(x-2)L_y^3 + 4(2x-1)L_y^2 + 4(x-2)L_s L_y^2\right.$$

$$+4(x-2)L_s^2 L_y + \left(8(x-1)-2(x-2)\pi^2\right)L_y + 8(x-2)\text{Li}_2(x)L_y + 8(2x-1)L_s L_y$$

$$+\left(-x+\frac{2}{1-x}-2\right)\pi^2 + 4\left(x-\frac{1}{1-x}+2\right)L_m^2 + 2\left(x-\frac{2}{1-x}+2\right)L_s^2$$

$$+8\left(-6x+(4x-6)\zeta_3-\frac{3}{1-x}+8\right) + 8(2x-1)\text{Li}_2(x) - 8(x-2)\text{Li}_3(x)$$

$$-8\left(x-\frac{2}{1-x}+4\right)L_m - 4\left(-13x-\frac{4}{1-x}+26\right)L_s + 8(x-2)\text{Li}_2(x)L_s$$

$$+8\left(x-\frac{1}{1-x}+2\right)L_m L_s + 8(x-2)S_{1,2}(x)\right]\right\} , \tag{A.3}$$

$$\mathcal{J}_3^{(0\times1)} = \left\{\frac{1}{m_s^2}\left[-8(1-x)x\right] + \frac{1}{m_s}\left[-8\left(4x^2-4x+3\right)\right] + \left[96\left(x^2-x+1\right)\right]\right\}$$

$$+\epsilon\left\{\frac{1}{m_s^2}\left[-16(1-x)x + 4(1-x)\zeta_3 x + 8(1-x)L_s x\right] + \frac{1}{m_s}\left[-16\left(4x^2-4x+1\right)\right.\right.$$

$$+4\left(4x^2-4x+3\right)\zeta_3 + 8\left(4x^2-4x+3\right)L_s\right] + \left[-16\left(-8x^2+8x+\left(3x^2-3x+3\right)\zeta_3-4\right)\right.$$

$$\left.\left.-96\left(x^2-x+1\right)L_s\right]\right\}$$

$$+\epsilon i\pi\left\{\frac{1}{m_s^2}\left[-8(1-x)x\right] + \frac{1}{m_s}\left[-8\left(4x^2-4x+3\right)\right] + \left[96\left(x^2-x+1\right)\right]\right\}$$

$$+\epsilon^2\left\{\frac{1}{m_s^2}\left[\frac{1}{15}(1-x)x\pi^4 + \frac{14}{3}(1-x)x\pi^2 - 4(1-x)xL_s^2 - 32(1-x)x + 16(1-x)xL_s\right.\right.$$

$$+\zeta_3(6(1-x)x - 4(1-x)xL_s)] + \frac{1}{m_s}\left[\frac{1}{15}\left(4x^2-4x+3\right)\pi^4 + \frac{14}{3}\left(4x^2-4x+3\right)\pi^2\right.$$

$$-4\left(4x^2-4x+3\right)L_s^2 - 32\left(4x^2-4x+1\right) + 16\left(4x^2-4x+1\right)L_s + \zeta_3\left(2\left(12x^2-12x+1\right)\right.$$

$$\left.-4\left(4x^2-4x+3\right)L_s\right)] + \left[-\frac{4}{5}\left(x^2-x+1\right)\pi^4 - 56\left(x^2-x+1\right)\pi^2 + 48\left(x^2-x+1\right)L_s^2\right.$$

$$-8\left(-32x^2+32x+\left(5x^2-5x+1\right)\zeta_3-16\right) + 16\left(-8x^2+8x+\left(3x^2-3x+3\right)\zeta_3-4\right)L_s\right]\right\}$$

$$+\epsilon^2 i\pi\left\{\frac{1}{m_s^2}\left[-16(1-x)x + 4(1-x)\zeta_3 x + 8(1-x)L_s x\right] + \frac{1}{m_s}\left[-16\left(4x^2-4x+1\right)\right.\right.$$

$$+4\left(4x^2-4x+3\right)\zeta_3 + 8\left(4x^2-4x+3\right)L_s\right] + \left[-16\left(-8x^2+8x+\left(3x^2-3x+3\right)\zeta_3-4\right)\right.$$

$$\left.\left.-96\left(x^2-x+1\right)L_s\right]\right\} . \tag{A.4}$$

References

[1] CDF collaboration, D.E. Acosta et al., *Measurement of the W^+W^- production cross section in $p\bar{p}$ collisions at $\sqrt{s} = 1.96\,TeV$ using dilepton events*, *Phys. Rev. Lett.* **94** (2005) 211801 [hep-ex/0501050].

[2] D0 collaboration, V.M. Abazov et al., *Measurement of the WW production cross section in $p\bar{p}$ collisions at $\sqrt{s} = 1.96\,TeV$*, *Phys. Rev. Lett.* **94** (2005) 151801 [*Erratum ibid.* **100** (2008) 139901] [hep-ex/0410066].

[3] M. Dittmar and H.K. Dreiner, *How to find a Higgs boson with a mass between $155\,GeV$ to $180\,GeV$ at the LHC*, *Phys. Rev.* **D 55** (1997) 167 [hep-ph/9608317].

[4] R.W. Brown and K.O. Mikaelian, *W^+W^- and Z^0Z^0 pair production in e^+e^-, pp, $p\bar{p}$ colliding beams*, *Phys. Rev.* **D 19** (1979) 922.

[5] J. Ohnemus, *An order α_s calculation of hadronic W^-W^+ production*, *Phys. Rev.* **D 44** (1991) 1403.

[6] S. Frixione, *A next-to-leading order calculation of the cross-section for the production of W^+W^- pairs in hadronic collisions*, *Nucl. Phys.* **B 410** (1993) 280.

[7] L.J. Dixon, Z. Kunszt and A. Signer, *Helicity amplitudes for $O(\alpha_s)$ production of W^+W^-, $W^\pm Z$, ZZ, $W^\pm\gamma$, or $Z\gamma$ pairs at hadron colliders*, *Nucl. Phys.* **B 531** (1998) 3 [hep-ph/9803250].

[8] L.J. Dixon, Z. Kunszt and A. Signer, *Vector boson pair production in hadronic collisions at $O(\alpha_s)$: lepton correlations and anomalous couplings*, *Phys. Rev.* **D 60** (1999) 114037 [hep-ph/9907305].

[9] J.M. Campbell and R.K. Ellis, *An update on vector boson pair production at hadron colliders*, *Phys. Rev.* **D 60** (1999) 113006 [hep-ph/9905386].

[10] M. Grazzini, *Soft-gluon effects in $W\,W$ production at hadron colliders*, *JHEP* **01** (2006) 095 [hep-ph/0510337].

[11] M. Spira, A. Djouadi, D. Graudenz and P.M. Zerwas, *Higgs boson production at the LHC*, *Nucl. Phys.* **B 453** (1995) 17 [hep-ph/9504378].

[12] S. Dawson, *Radiative corrections to Higgs boson production*, *Nucl. Phys.* **B 359** (1991) 283.

[13] R.V. Harlander and W.B. Kilgore, *Next-to-next-to-leading order Higgs production at hadron colliders*, *Phys. Rev. Lett.* **88** (2002) 201801 [hep-ph/0201206].

[14] C. Anastasiou and K. Melnikov, *Higgs boson production at hadron colliders in NNLO QCD*, *Nucl. Phys.* **B 646** (2002) 220 [hep-ph/0207004].

[15] V. Ravindran, J. Smith and W.L. van Neerven, *NNLO corrections to the total cross section for Higgs boson production in hadron hadron collisions*, *Nucl. Phys.* **B 665** (2003) 325 [hep-ph/0302135].

[16] S. Catani, D. de Florian and M. Grazzini, *Direct Higgs production and jet veto at the Tevatron and the LHC in NNLO QCD*, *JHEP* **01** (2002) 015 [hep-ph/0111164].

[17] G. Davatz, G. Dissertori, M. Dittmar, M. Grazzini and F. Pauss, *Effective K-factors for $gg \to H \to WW \to \ell\nu\ell\nu$ at the LHC*, *JHEP* **05** (2004) 009 [hep-ph/0402218].

[18] C. Anastasiou, K. Melnikov and F. Petriello, *Higgs boson production at hadron colliders: differential cross sections through next-to-next-to-leading order*, *Phys. Rev. Lett.* **93** (2004) 262002 [`hep-ph/0409088`].

[19] C. Anastasiou, G. Dissertori and F. Stockli, *NNLO QCD predictions for the $H \to WW \to \ell\ell\nu\nu$ signal at the LHC*, *JHEP* **09** (2007) 018 [`arXiv:0707.2373`].

[20] M. Grazzini, *NNLO predictions for the Higgs boson signal in the $H \to WW \to \ell\nu\ell\nu$ and $H \to ZZ \to 4\ell$ decay channels*, *JHEP* **02** (2008) 043 [`arXiv:0801.3232`].

[21] A. Bredenstein, A. Denner, S. Dittmaier and M.M. Weber, *Precise predictions for the Higgs-boson decay $H \to WW/ZZ \to 4$ leptons*, *Phys. Rev.* **D 74** (2006) 013004 [`hep-ph/0604011`].

[22] T. Binoth, M. Ciccolini, N. Kauer and M. Krämer, *Gluon-induced WW background to Higgs boson searches at the LHC*, *JHEP* **03** (2005) 065 [`hep-ph/0503094`].

[23] T. Binoth, M. Ciccolini, N. Kauer and M. Krämer, *Gluon-induced W-boson pair production at the LHC*, *JHEP* **12** (2006) 046 [`hep-ph/0611170`].

[24] C. Anastasiou, E.W.N. Glover and M.E. Tejeda-Yeomans, *Two-loop QED and QCD corrections to massless fermion boson scattering*, *Nucl. Phys.* **B 629** (2002) 255 [`hep-ph/0201274`].

[25] M. Czakon, A. Mitov and S. Moch, *Heavy-quark production in massless quark scattering at two loops in QCD*, *Phys. Lett.* **B 651** (2007) 147 [`arXiv:0705.1975`].

[26] M. Czakon, A. Mitov and S. Moch, *Heavy-quark production in gluon fusion at two loops in QCD*, *Nucl. Phys.* **B 798** (2008) 210 [`arXiv:0707.4139`].

[27] M. Czakon, J. Gluza and T. Riemann, *Master integrals for massive two-loop Bhabha scattering in QED*, *Phys. Rev.* **D 71** (2005) 073009 [`hep-ph/0412164`].

[28] M. Czakon, J. Gluza and T. Riemann, *The planar four-point master integrals for massive two-loop Bhabha scattering*, *Nucl. Phys.* **B 751** (2006) 1 [`hep-ph/0604101`].

[29] S. Actis, M. Czakon, J. Gluza and T. Riemann, *Two-loop fermionic corrections to massive Bhabha scattering*, *Nucl. Phys.* **B 786** (2007) 26 [`arXiv:0704.2400`].

[30] S. Laporta, *High-precision calculation of multi-loop Feynman integrals by difference equations*, *Int. J. Mod. Phys.* **A 15** (2000) 5087 [`hep-ph/0102033`].

[31] V.A. Smirnov, *Analytical result for dimensionally regularized massless on-shell double box*, *Phys. Lett.* **B 460** (1999) 397 [`hep-ph/9905323`].

[32] J.B. Tausk, *Non-planar massless two-loop Feynman diagrams with four on-shell legs*, *Phys. Lett.* **B 469** (1999) 225 [`hep-ph/9909506`].

[33] G. Chachamis and M. Czakon, *MBrepresentation.m*, unpublished.

[34] M. Czakon, *Automatized analytic continuation of Mellin-Barnes integrals*, *Comput. Phys. Commun.* **175** (2006) 559 [`hep-ph/0511200`].

[35] S. Moch and P. Uwer, *XSummer: transcendental functions and symbolic summation in form*, *Comput. Phys. Commun.* **174** (2006) 759 [`math-ph/0508008`].

[36] H.R.P. Ferguson and D.H. Bailey, *A polynomial time, numerically stable integer relation algorithm*, RNR technical report RNR-91-032 (1992), (see e.g. http://mathworld.wolfram.com/PSLQAlgorithm.html).

[37] S. Catani, *The singular behaviour of QCD amplitudes at two-loop order*, *Phys. Lett.* **B 427** (1998) 161 [`hep-ph/9802439`].

[38] T. van Ritbergen, J.A.M. Vermaseren and S.A. Larin, *The four-loop β-function in quantum chromodynamics*, *Phys. Lett.* **B 400** (1997) 379 [`hep-ph/9701390`].

[39] M. Czakon, *The four-loop QCD β-function and anomalous dimensions*, *Nucl. Phys.* **B 710** (2005) 485 [`hep-ph/0411261`].

[40] K.P.O. Diener, B.A. Kniehl and A. Pilaftsis, *Loop effects of exotic leptons on vector boson pair production at* e^+e^- *colliders*, *Phys. Rev.* **D 57** (1998) 2771 [`hep-ph/9709361`].

[41] A. Denner and T. Sack, *Electroweak radiative corrections to* $e^+e^- \to Z^0Z^0$, *Nucl. Phys.* **B 306** (1988) 221.

[42] G. Chachamis, *NNLO virtual corrections to* W^+W^- *production at the LHC*, *Acta Phys. Polon.* **B38** (2007) 3563 [`arXiv:0710.3035`].

[43] W.T. Giele and E.W.N. Glover, *Higher order corrections to jet cross-sections in* e^+e^- *annihilation*, *Phys. Rev.* **D 46** (1992) 1980.

[44] Z. Kunszt, A. Signer and Z. Trócsányi, *Singular terms of helicity amplitudes at one loop in QCD and the soft limit of the cross-sections of multiparton processes*, *Nucl. Phys.* **B 420** (1994) 550 [`hep-ph/9401294`].

[45] S. Catani and M.H. Seymour, *A general algorithm for calculating jet cross sections in NLO QCD*, *Nucl. Phys.* **B 485** (1997) 291 [*Erratum ibid.* **B 510** (1998) 503] [`hep-ph/9605323`].

[46] J.M. Campbell, R. Keith Ellis and G. Zanderighi, *Next-to-leading order predictions for* $WW + 1$ *jet distributions at the LHC*, *JHEP* **12** (2007) 056 [`arXiv:0710.1832`].

[47] S. Dittmaier, S. Kallweit and P. Uwer, *NLO QCD corrections to WW+jet production at hadron colliders*, *Phys. Rev. Lett.* **100** (2008) 062003 [`arXiv:0710.1577`].

[48] S. Catani and M. Grazzini, *An NNLO subtraction formalism in hadron collisions and its application to Higgs boson production at the LHC*, *Phys. Rev. Lett.* **98** (2007) 222002 [`hep-ph/0703012`].